

# Computational Modeling of Social Behavior

Day 4

## **Networks, etc.**

Paul Smaldino

# Outline of the day

- **Morning**

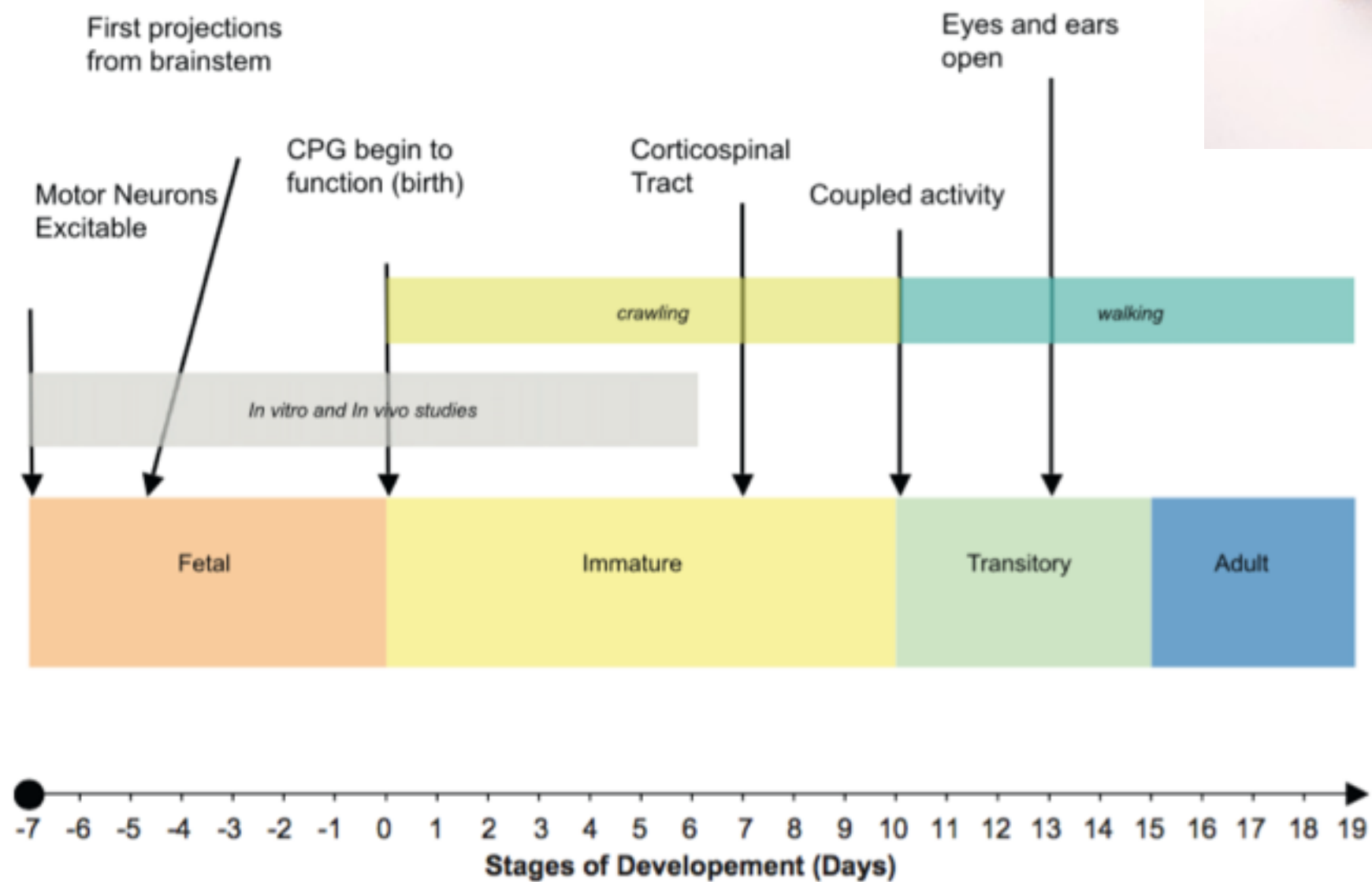
- Models and Empirical Data
- Network Theory

- **Afternoon**

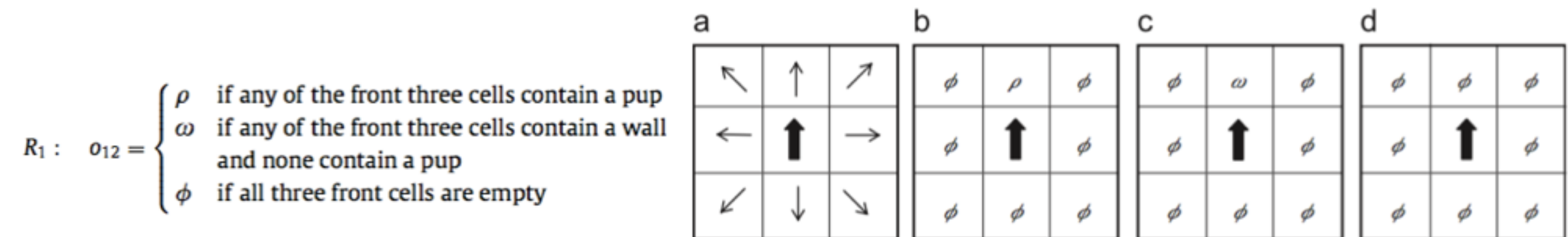
- Modeling Agents on Networks
- Coda: Why Model

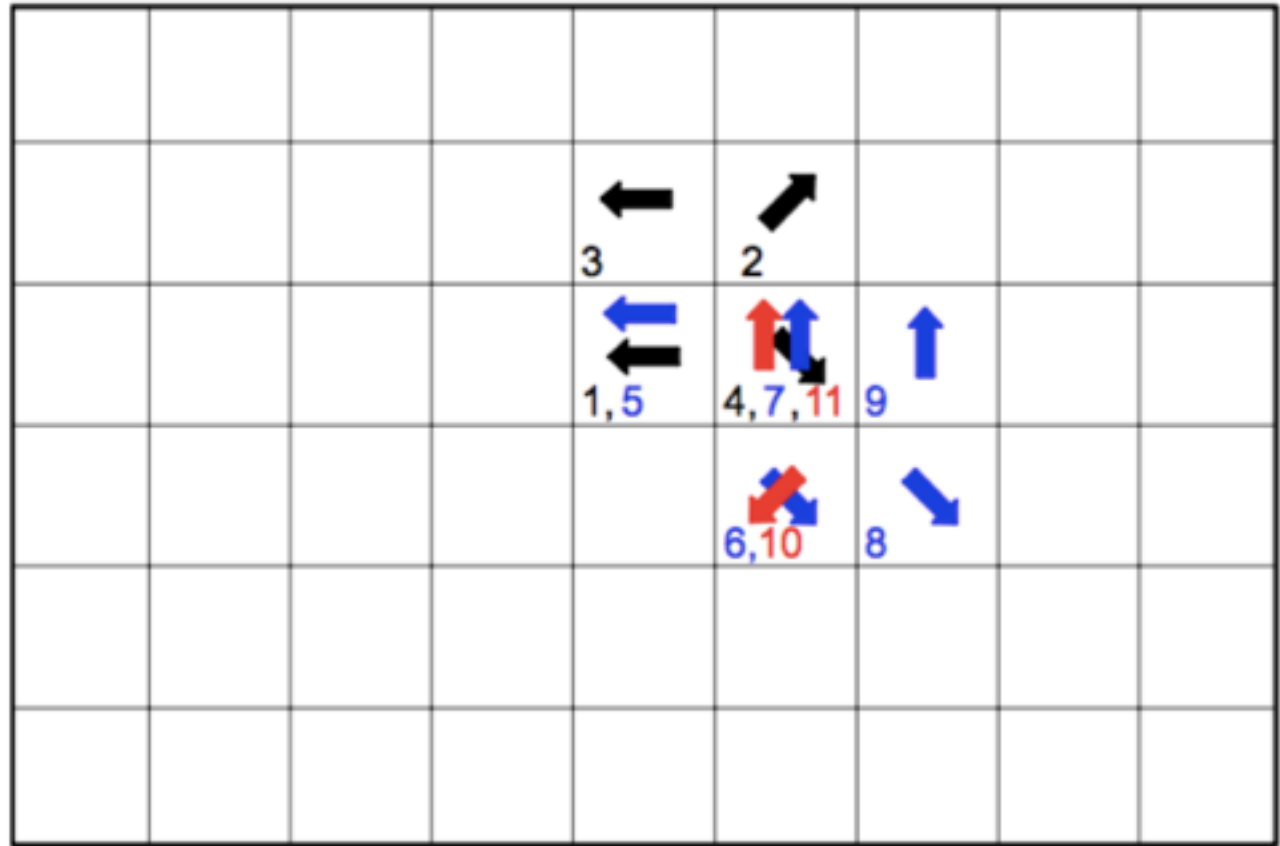
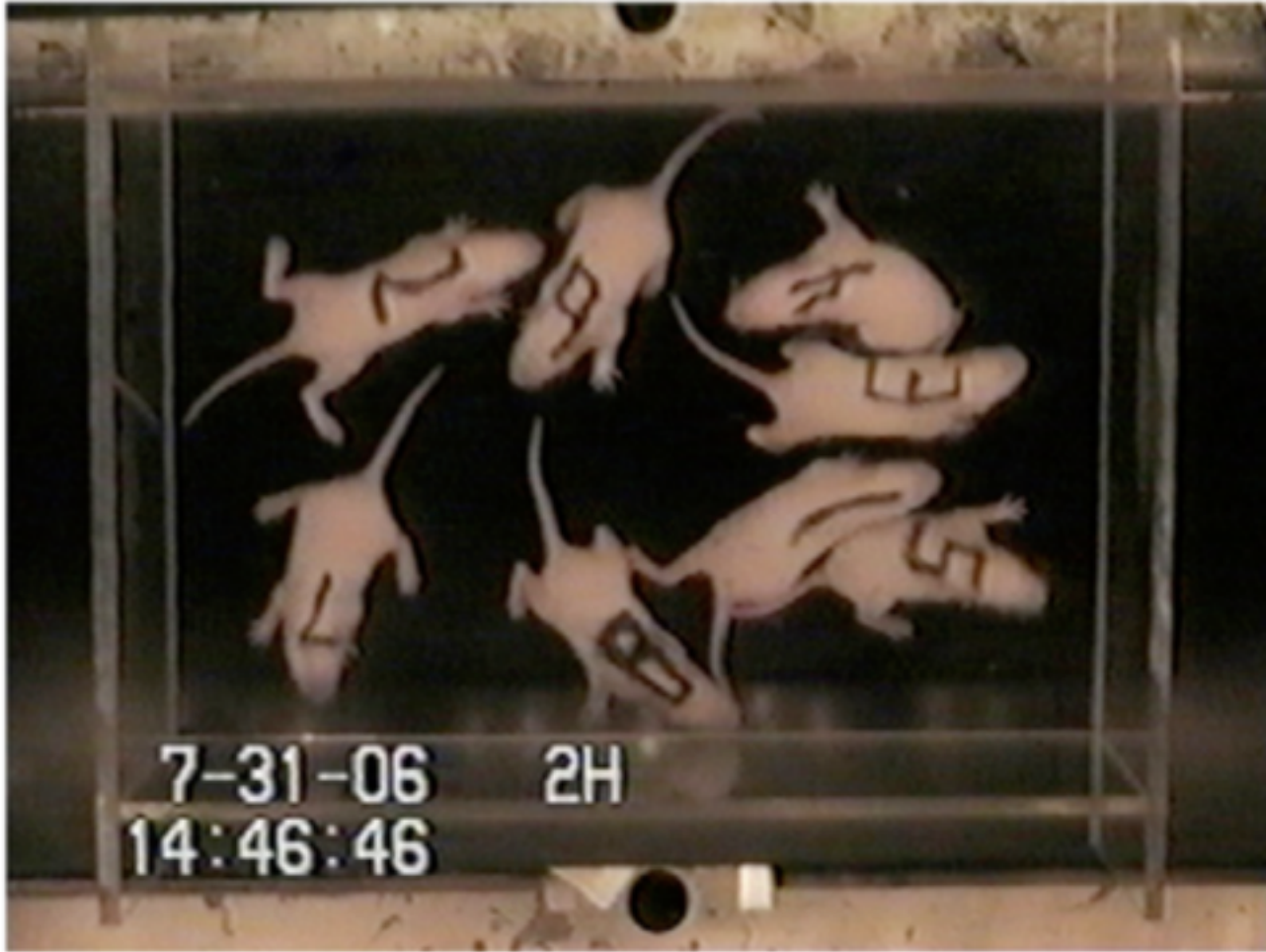
# What can we do with models?

- Scaffold theory development by creating mental models
- Explain generative mechanism behind existing data
- Predict future data

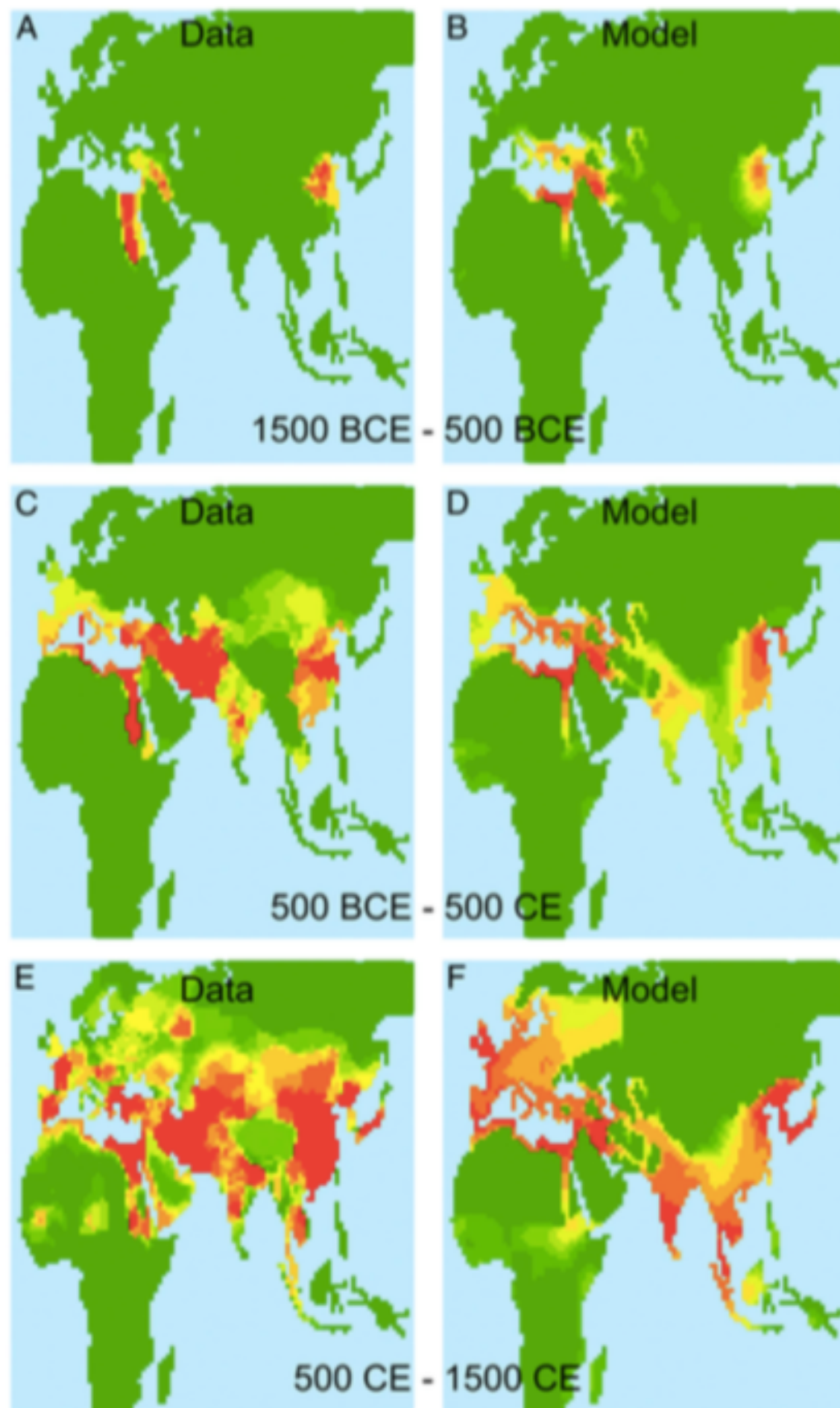


- Data collection:
  - Rat pups moved around in arena individually and in groups at 7 and 10 days old. Video capture.
- Model:
  - Agents move through simulated arena,
  - Evolved contingent movement behaviors in response to nothing, wall, and other pups





- Results:
  - ▶ Evolved models fit data than any null model
  - ▶ At 7 days, individual-evolved model fit group data with other pups treated as wall
  - ▶ At 10 days, individual-evolved model was terrible fit, required social contingent movement.
  - ▶ Supports conclusion that social awareness is not present at 7 days old, but is by 10 days old.



- Epstein: If you didn't generate it, you didn't explain it
- But, if you *did* generate it, you have only generated a *candidate* explanation





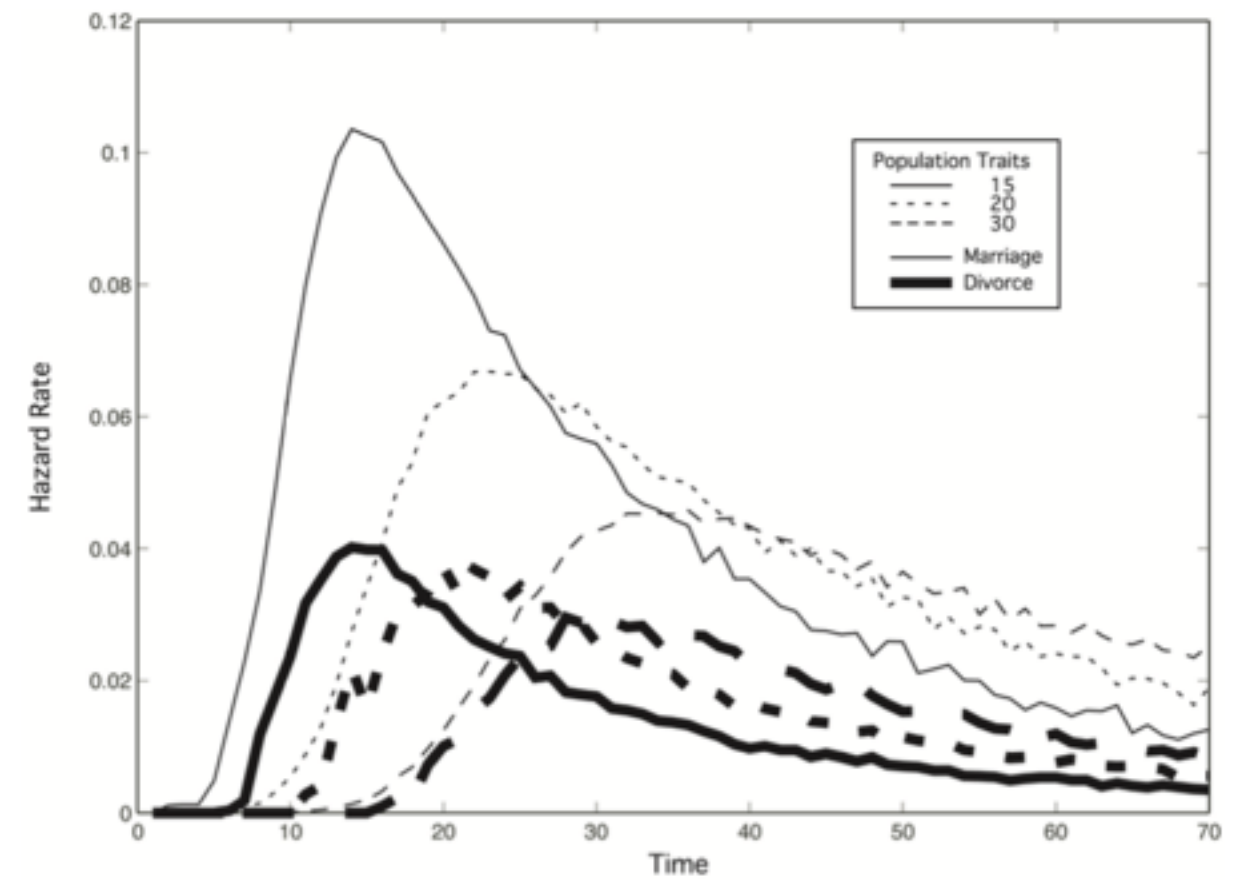
**ALIENS**

**H**HD  
HISTORY.COM

## Data



## Model



# Mate choice model

Male and female agents vary in “attractiveness” on 1-10 scale and have opportunities to form pairs.

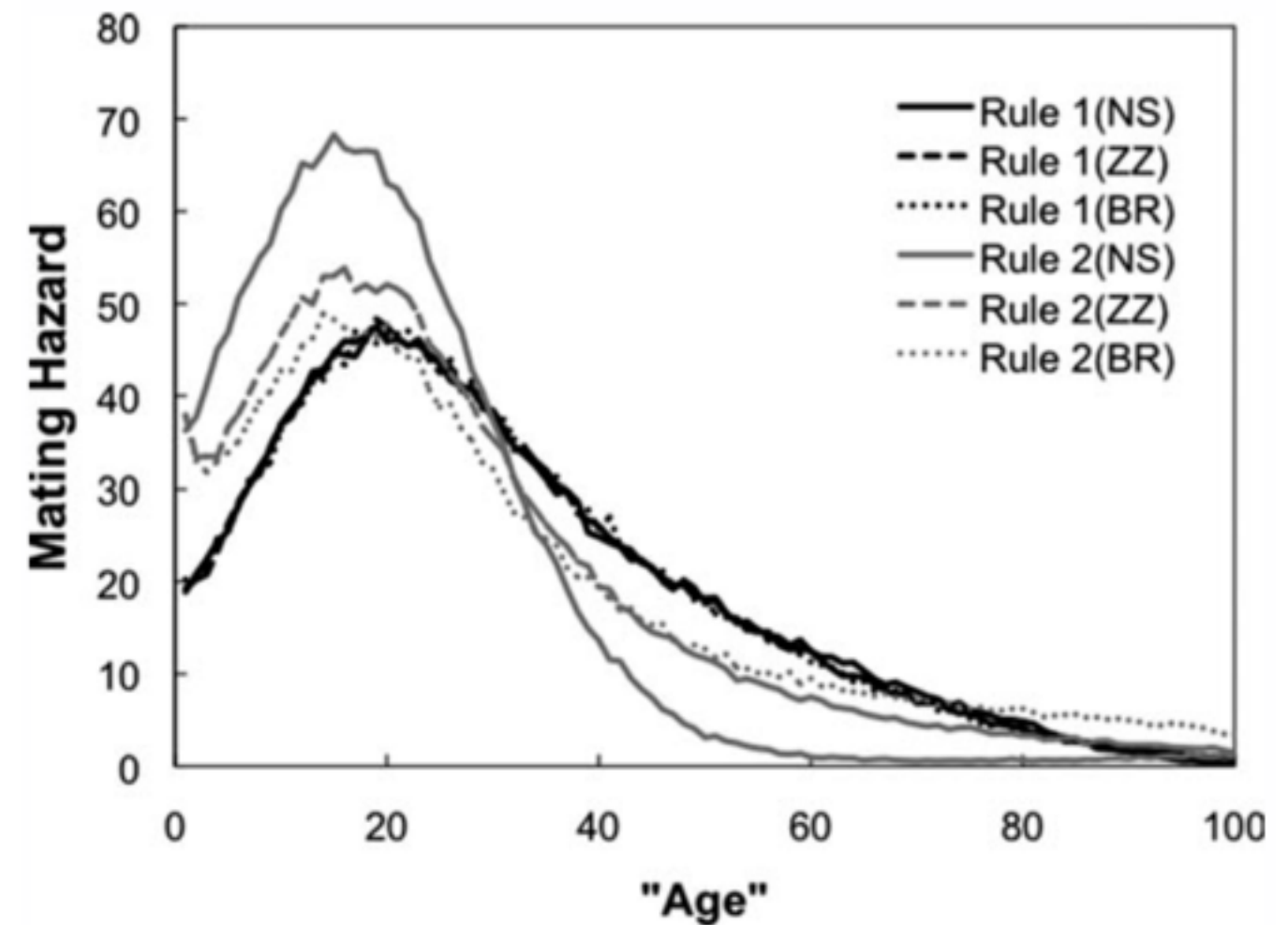
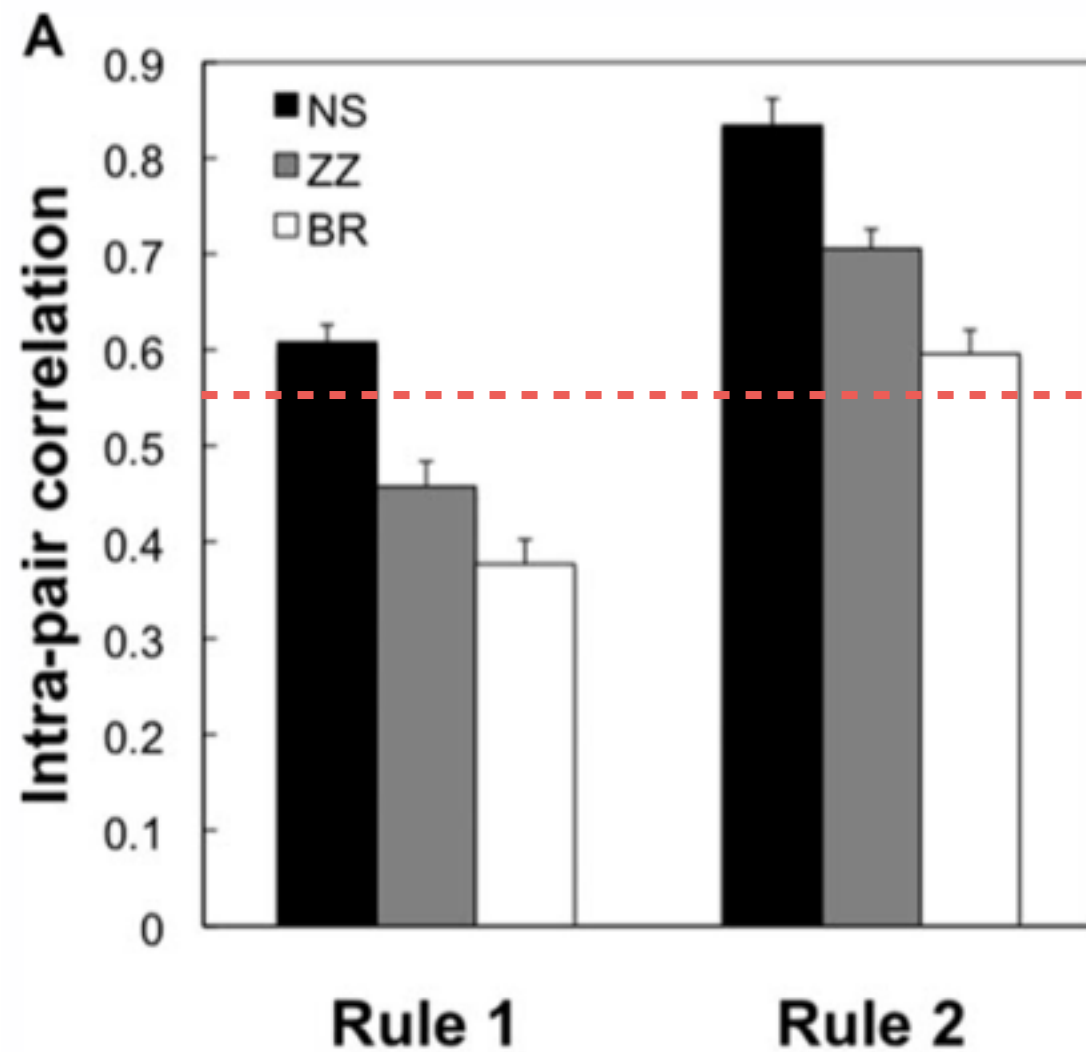
## Two decision rules:

1. Prefer the most attractive
2. Prefer the most similar

## Three movement rules

1. Non-spatial/well-mixed (NS)
2. Zigzag (ZZ): move rapidly through space
3. Brownian (BR): move slowly through space

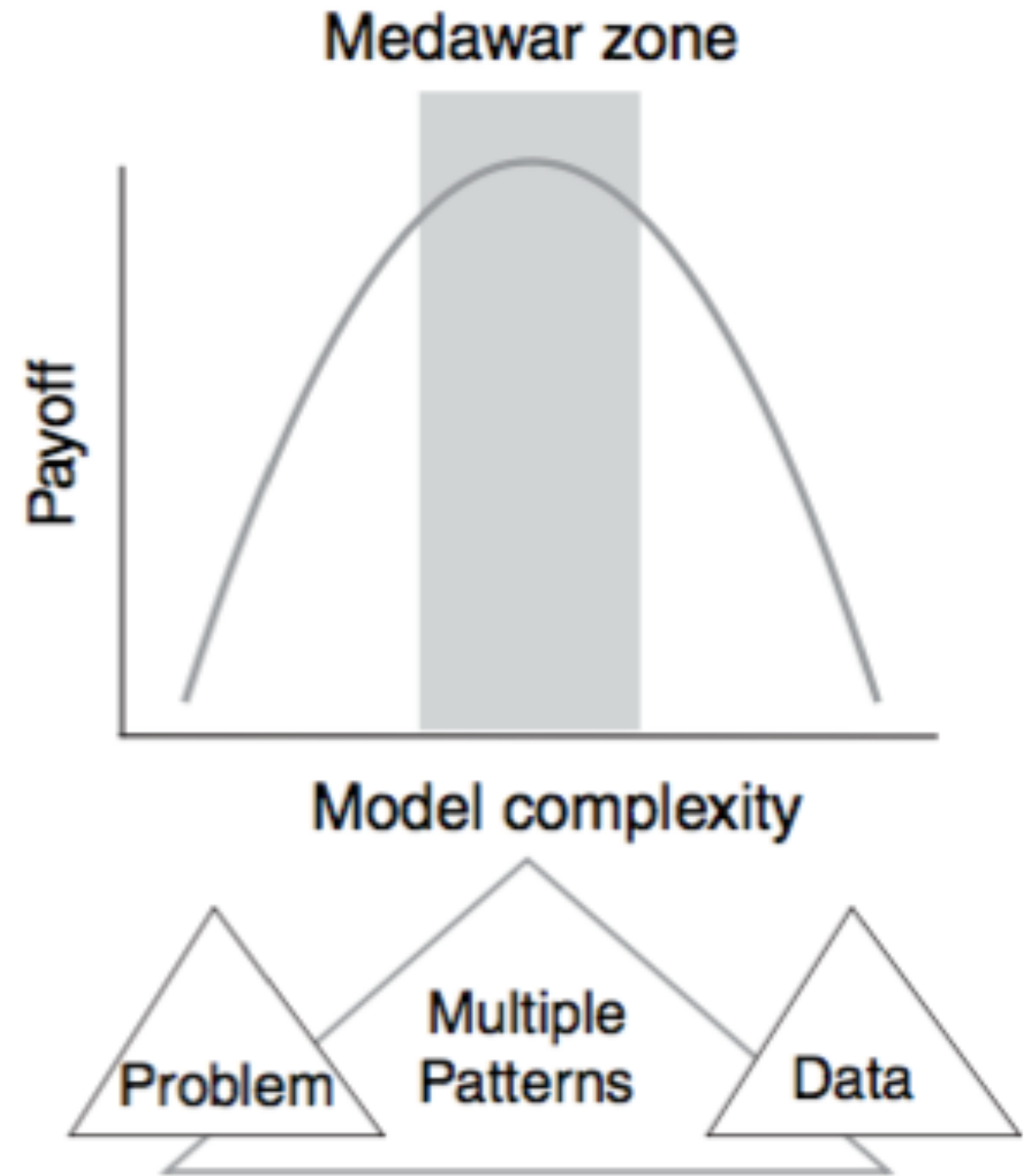
Assumptions about interaction networks can make two very different decision rules each fit the data





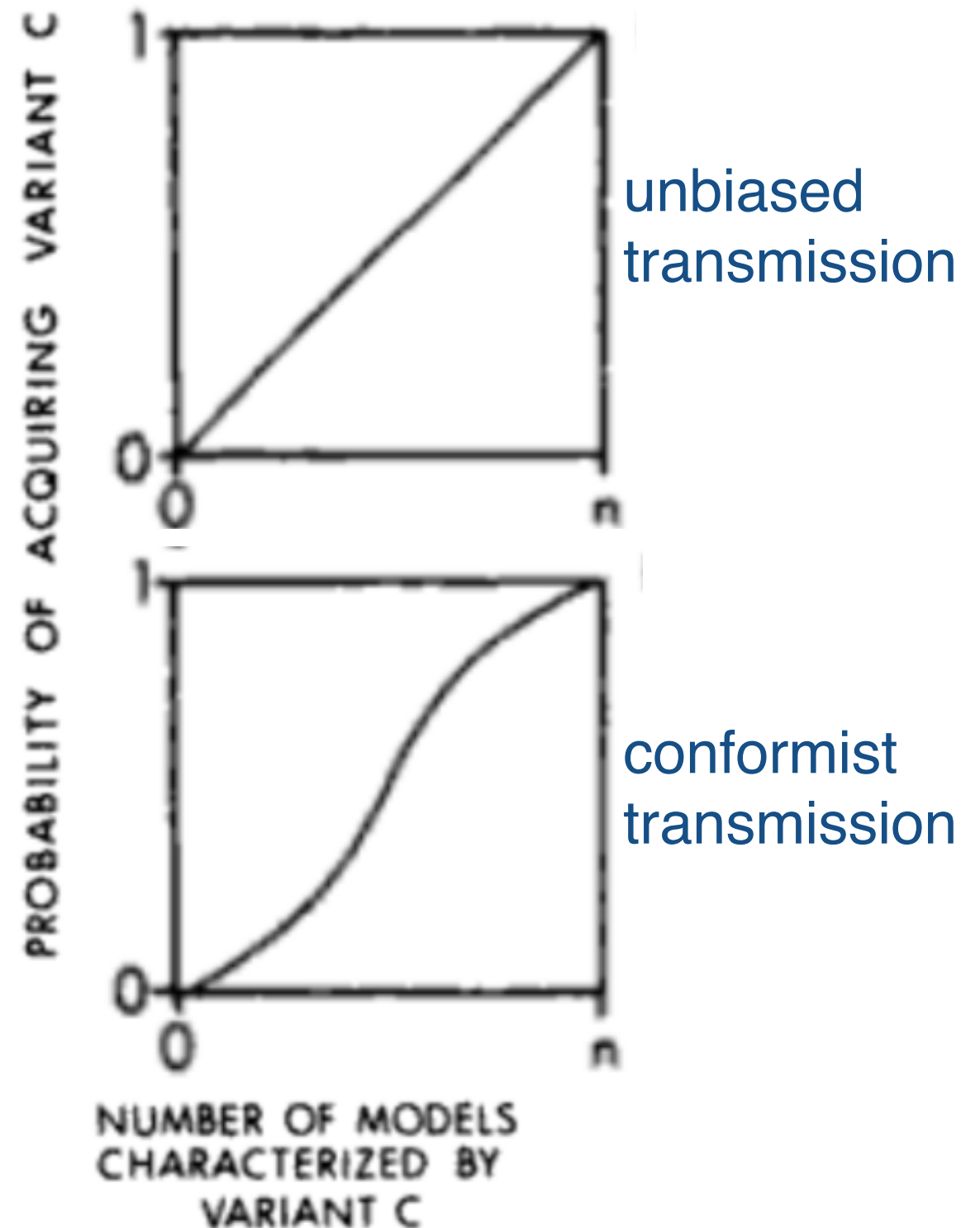
# Pattern-Oriented Modeling of Agent-Based Complex Systems: Lessons from Ecology

Volker Grimm,<sup>1\*</sup> Eloy Revilla,<sup>2</sup> Uta Berger,<sup>3</sup> Florian Jeltsch,<sup>4</sup> Wolf M. Mooij,<sup>5</sup> Steven F. Railsback,<sup>6</sup>  
Hans-Hermann Thulke,<sup>1</sup> Jacob Weiner,<sup>7</sup> Thorsten Wiegand,<sup>1</sup> Donald L. DeAngelis<sup>8</sup>



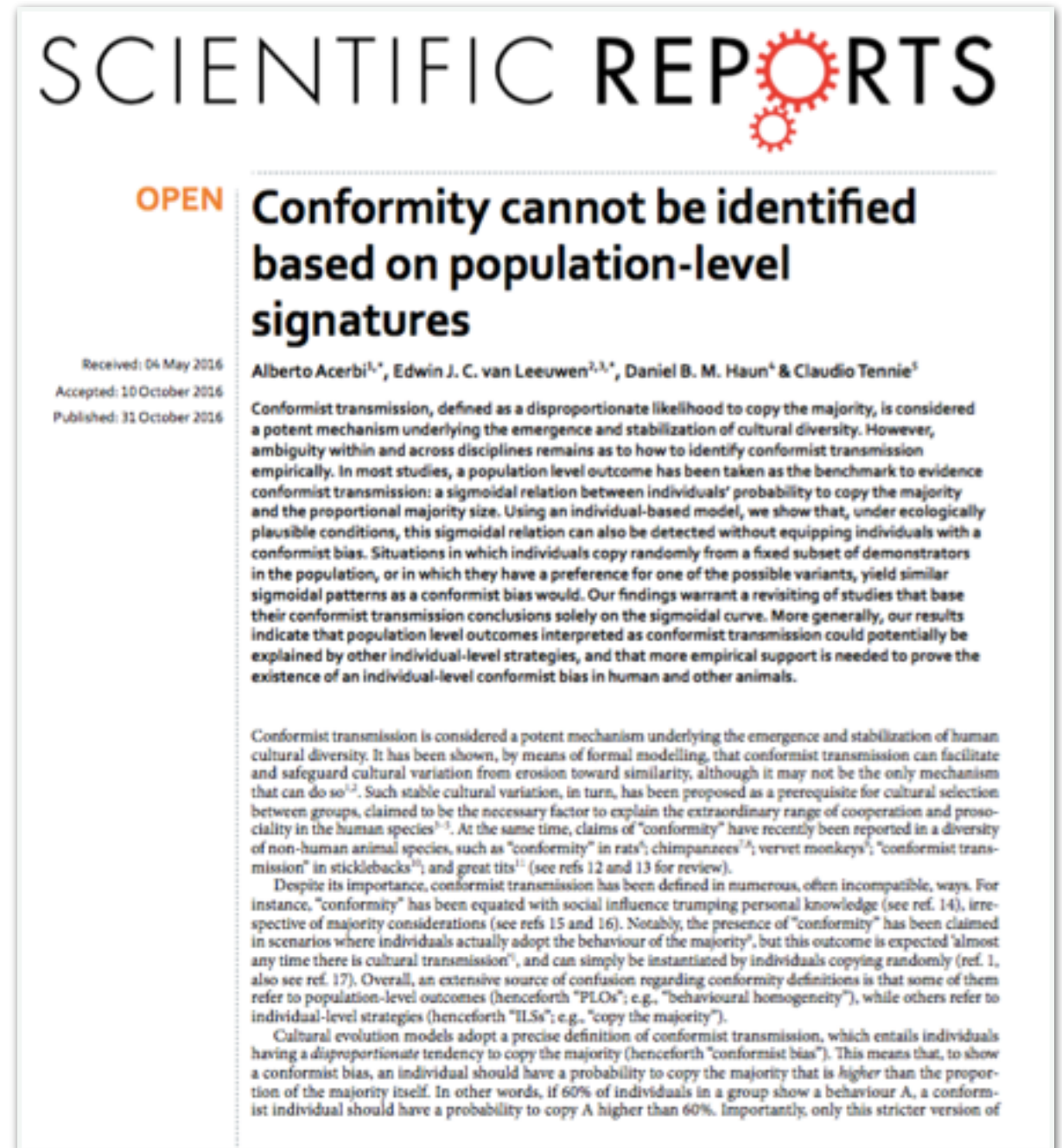
# Model assumptions are important

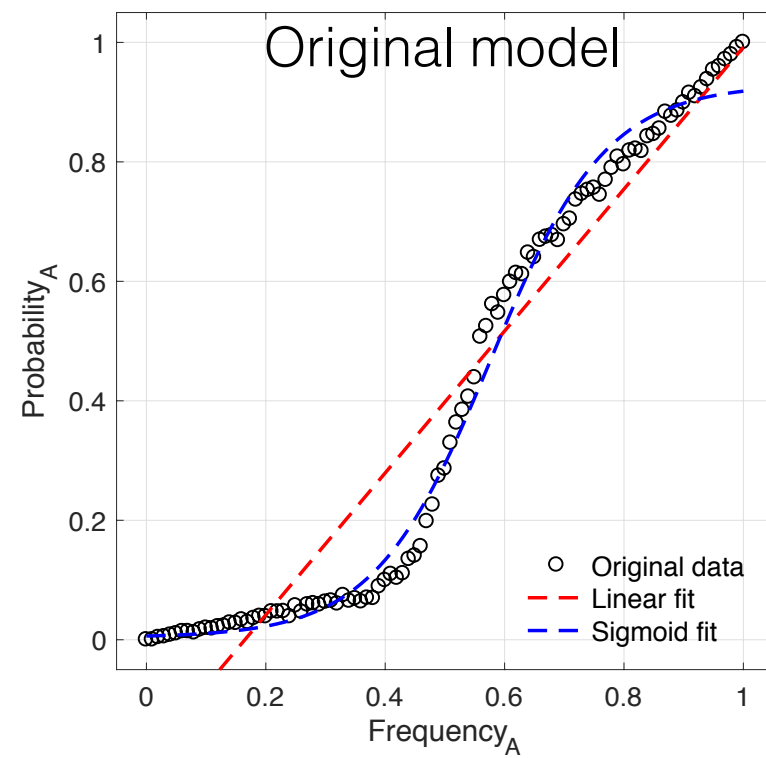
- Conformity: an above-baseline probability of adopting the common behavioral variant



# Model assumptions are important

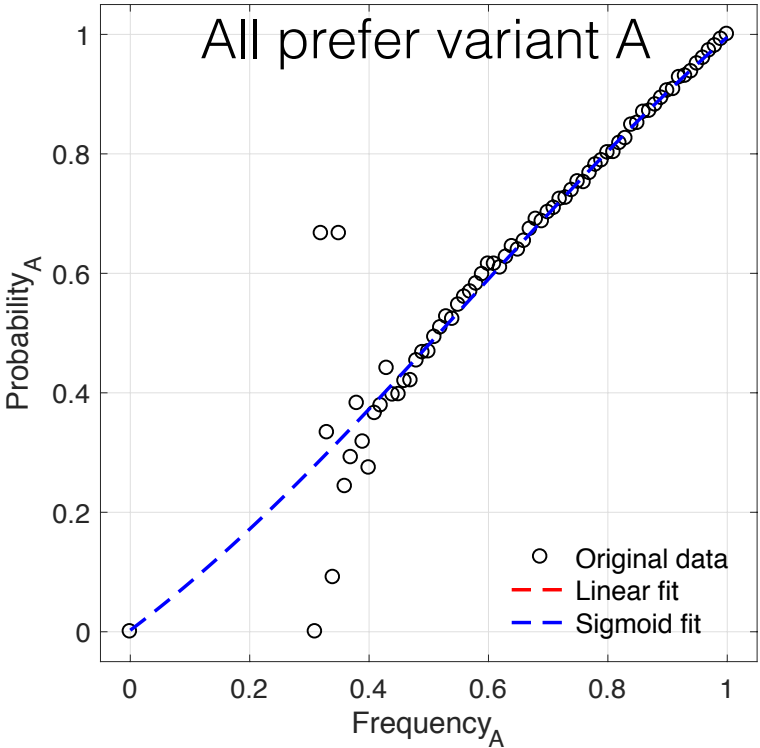
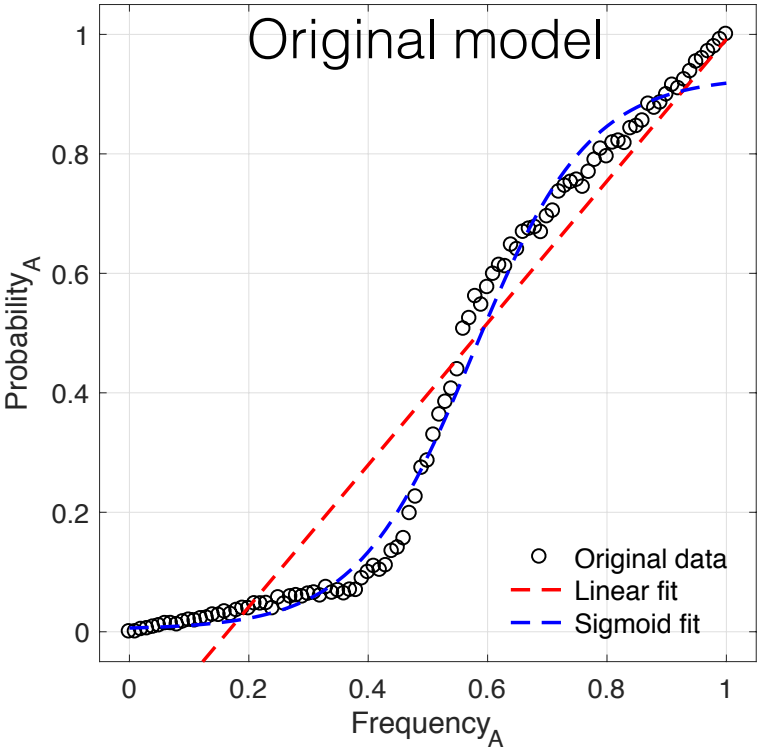
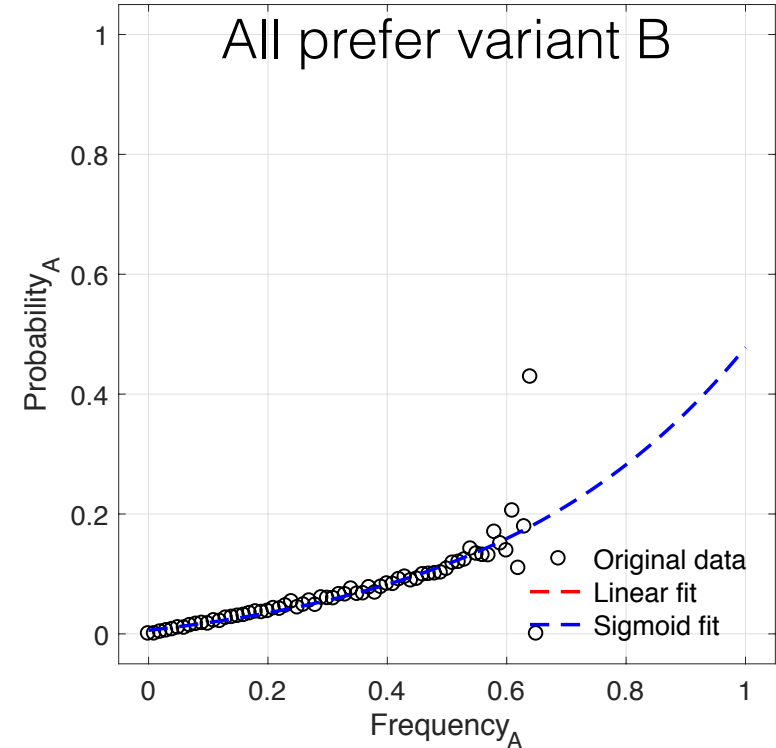
- Two variants: A and B
- Initial: 50% each
- In each run, one variant was preferred by all (direct bias)
- Each time step:
  - Each individual paired with randomly chosen demonstrator
  - If demonstrator had preferred variant, copy
  - Else, copy with probability  $p_{Less} = 0.2$

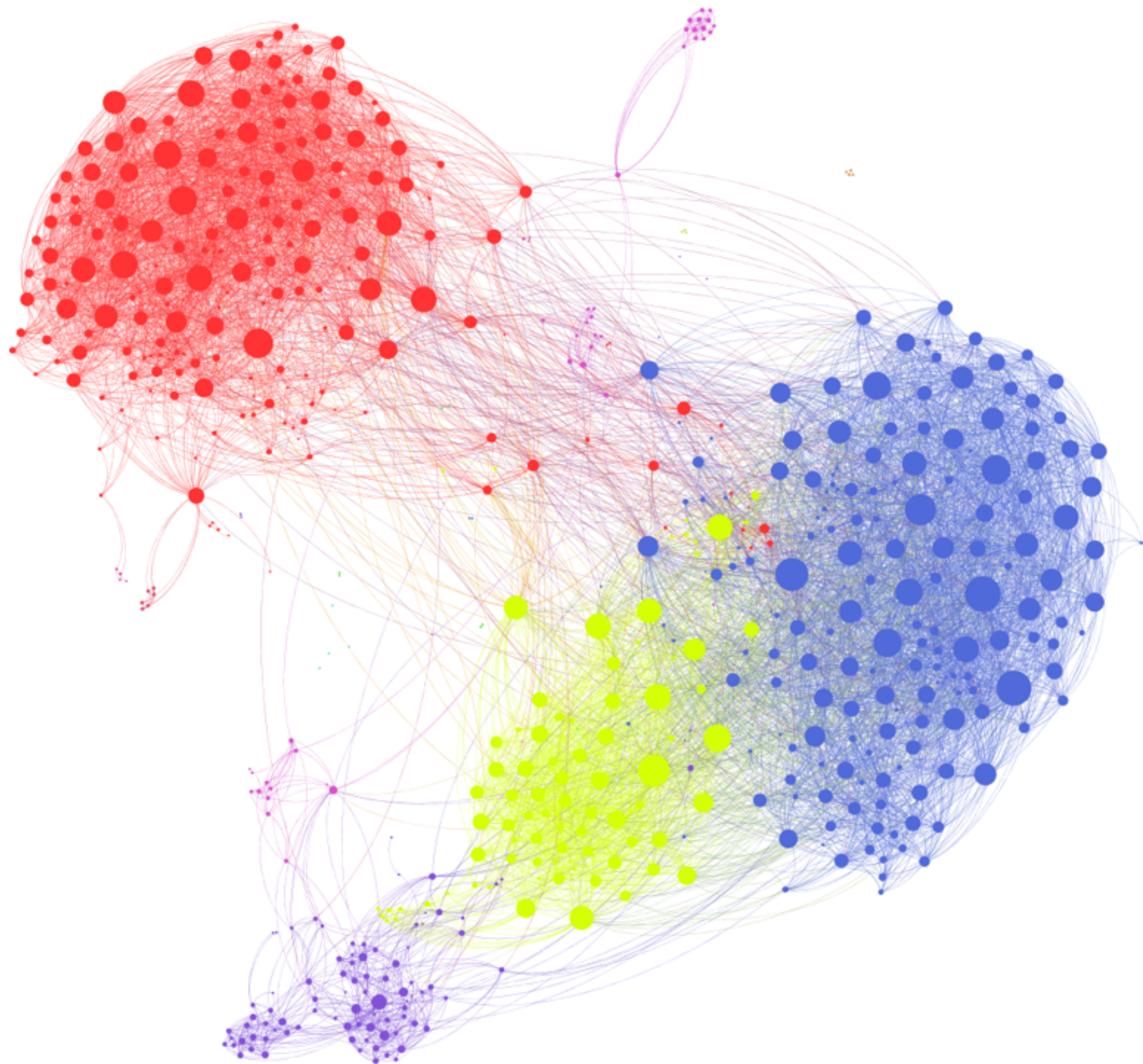






Initial frequency<sub>A</sub> always 50%



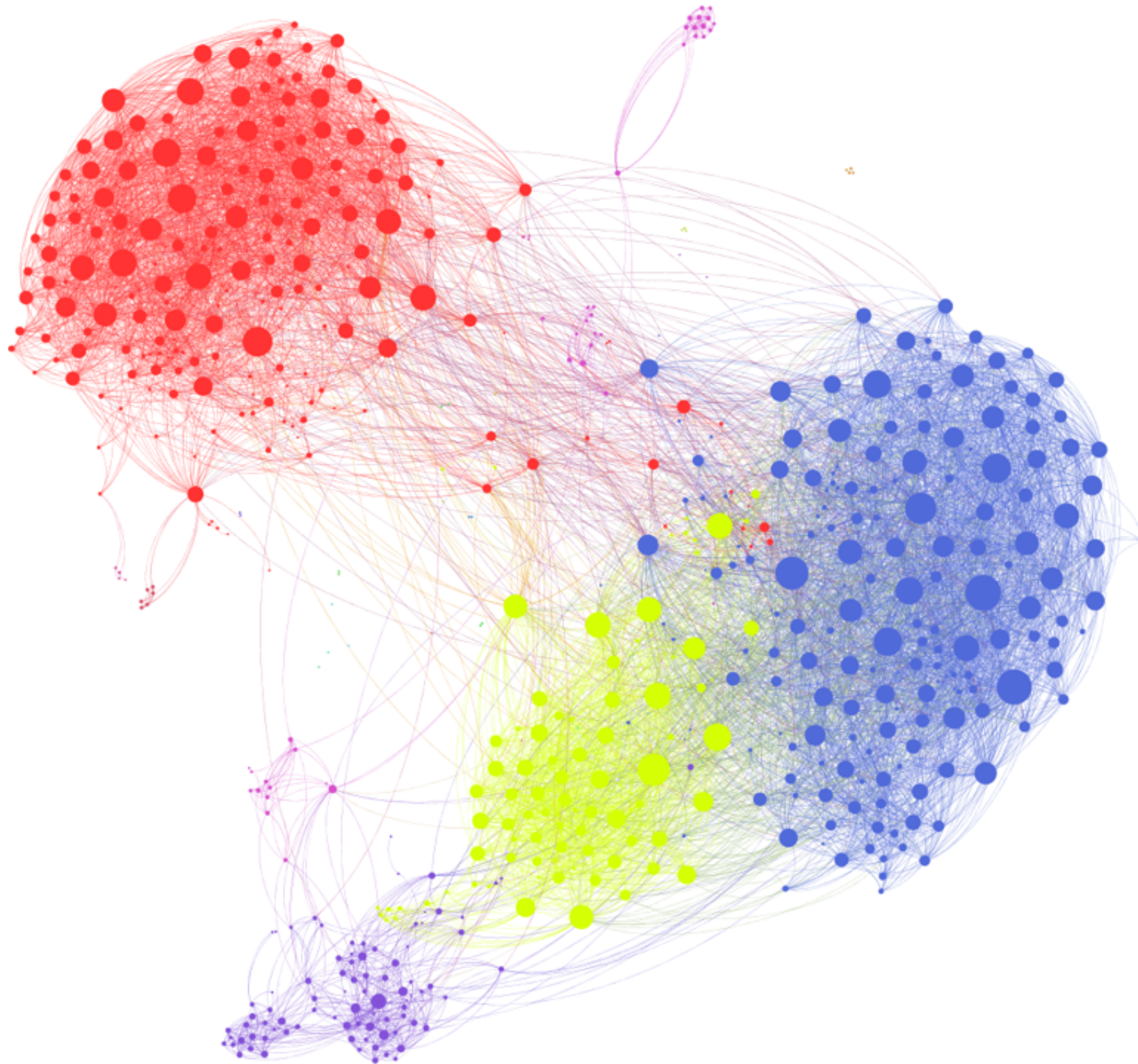




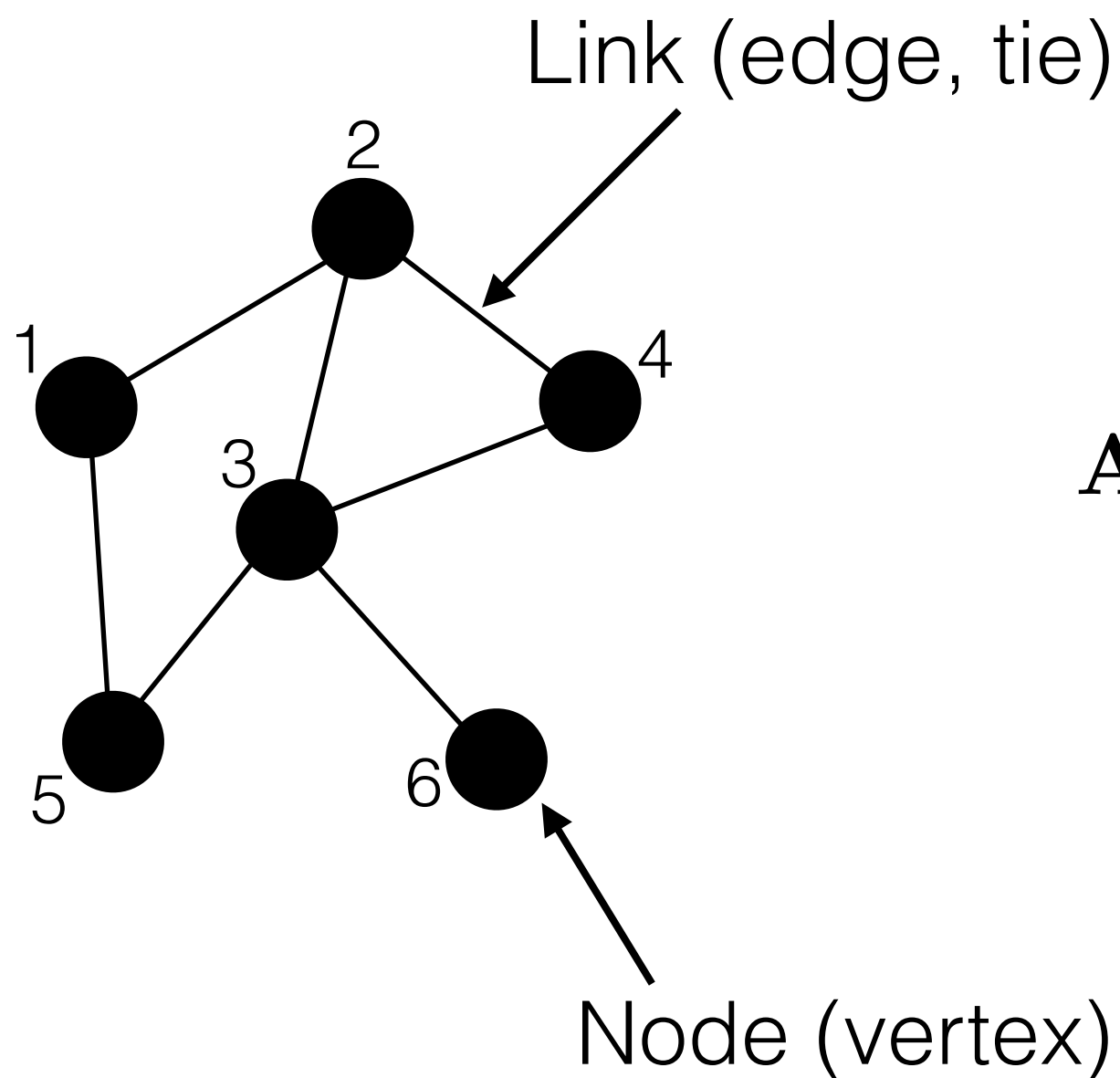




# Networks



# What is a network?

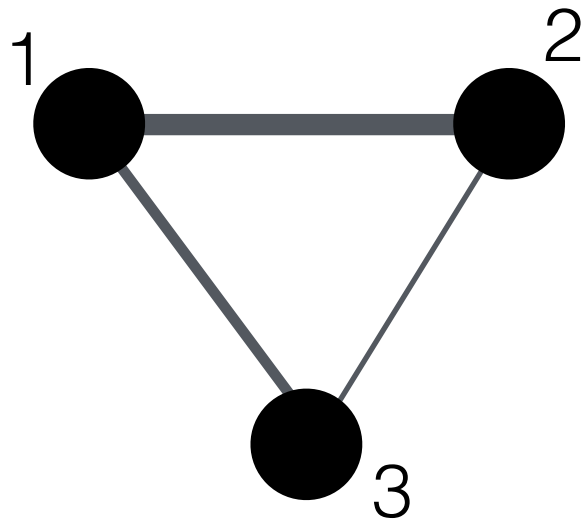


Adjacency matrix:

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

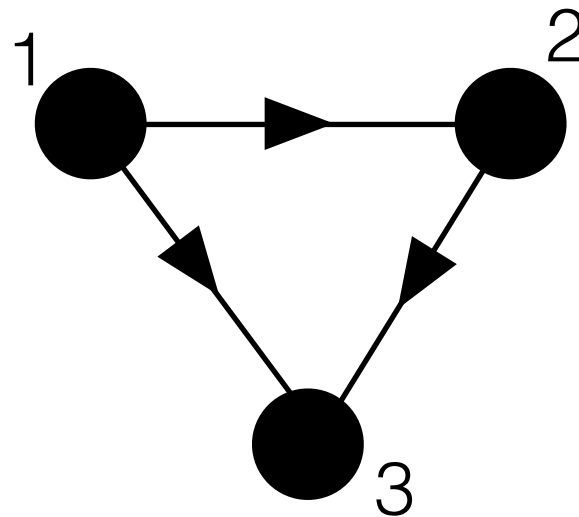
## Weighted networks

$$\mathbf{A} = \begin{pmatrix} 0 & 2 & 1 \\ 2 & 0 & 0.5 \\ 1 & 0.5 & 0 \end{pmatrix}$$

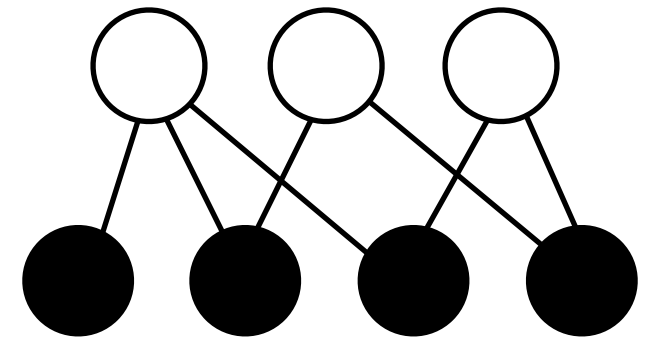


## Directed networks

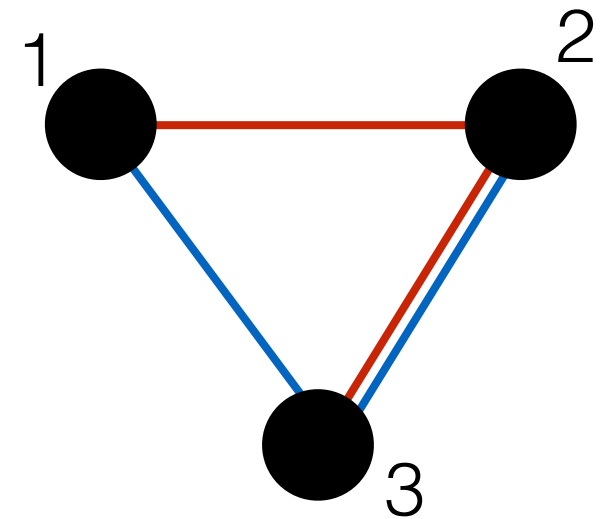
$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$



## Bipartite networks

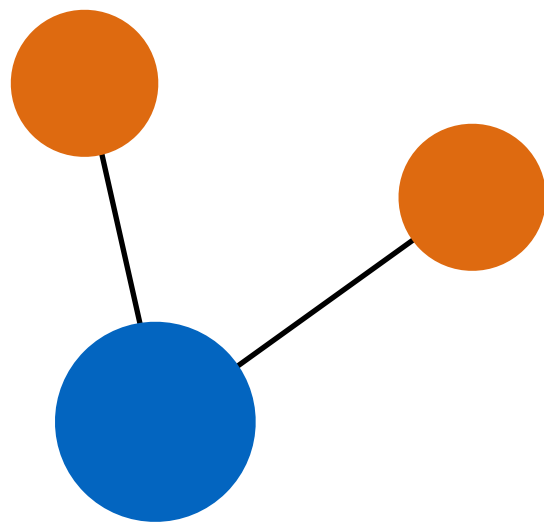


## Multiplex networks



# Degree and density

- Which are the most important or central nodes in a network?



$$k = 2$$

**degree:**  $k_i = \sum_{j=1}^n A_{ij}$

(also called 'degree centrality')

The **density** of a network is the proportion of possible edges that actually exist.

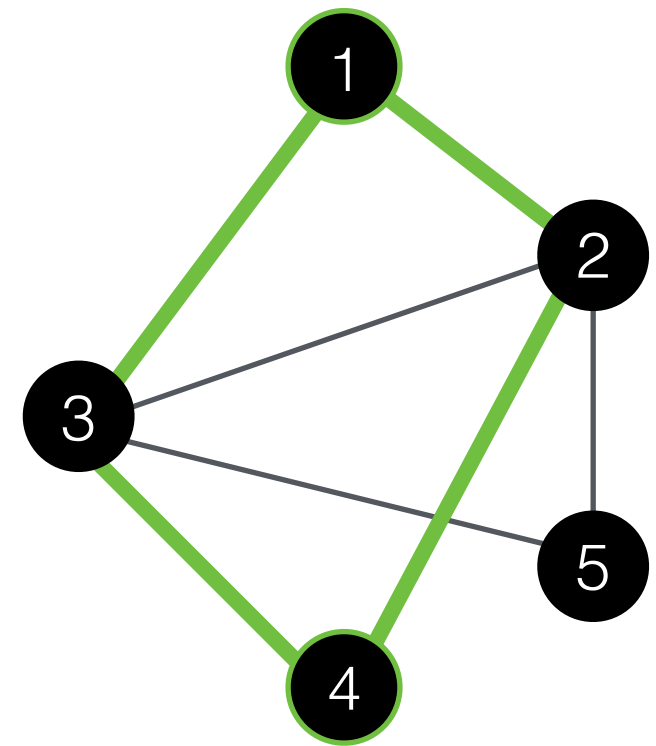
# Eigenvector Centrality

- Give more weight to edges that connect to highly-connected nodes
- Requires computing the eigenvectors of the adjacency matrix (requires linear algebra)
- **Google's** PageRank algorithm is a variant of this

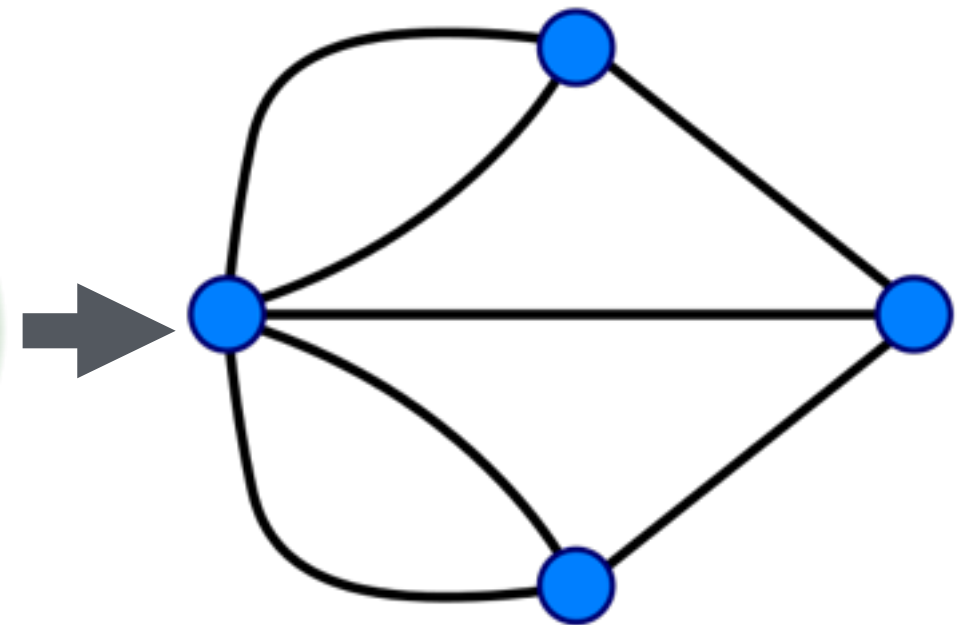
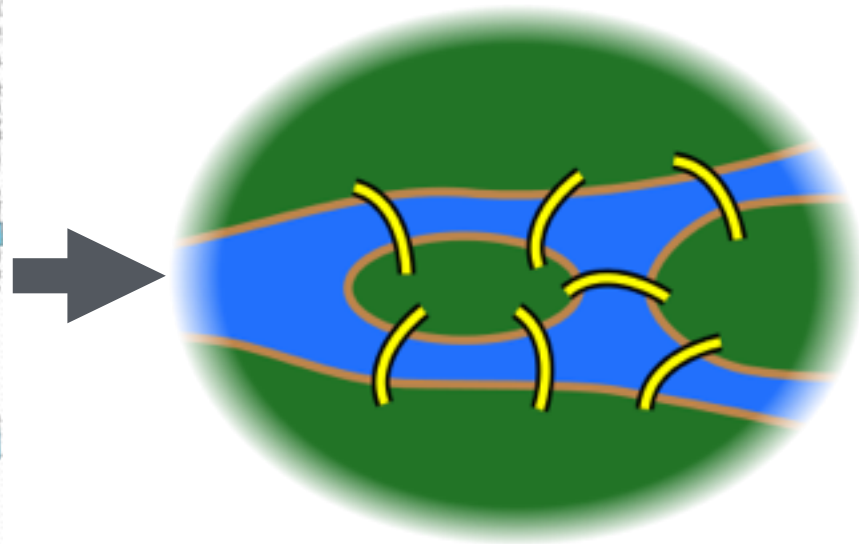
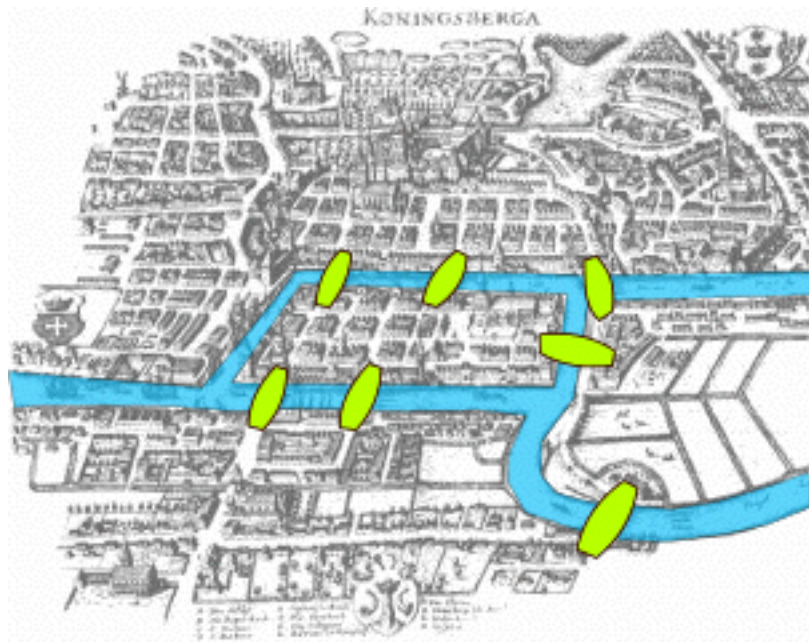


# Paths

- A **path** between two nodes is any sequence of non-repeating connected nodes that connects the two nodes
- The **shortest path** between two nodes is one that connects the two nodes with the smallest number of edges (also called the **distance** between the nodes)
- The **average path length** is the average distance between all pairs of nodes in a network



# Euler and the Seven Bridges of Königsberg

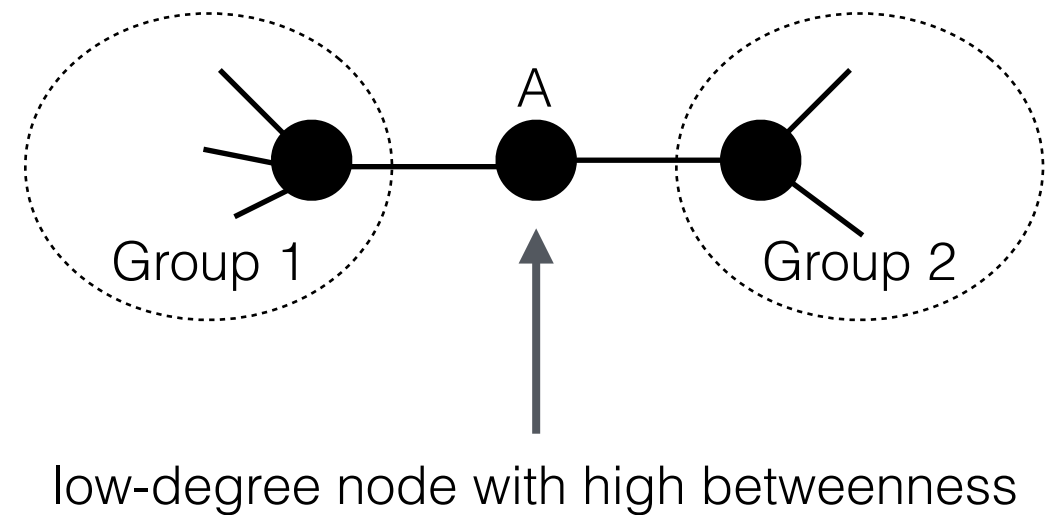


Is there any walking route that crosses all seven bridges exactly once?

# Betweenness Centrality

- Extent to which a node lies on paths between other nodes
- Let  $n_{st}^i$  be 1 if node  $i$  lies on the shortest path from node  $s$  to node  $t$ , and 0 if it doesn't (or if there is no such path). The betweenness centrality of node  $i$  is:

$$x_i = \sum_{st} n_{st}^i$$



# Closeness Centrality

- Based on mean distance from a node to other nodes.
- Take reciprocal so higher values indicate higher closeness

Mean distance from node  $i$   
to all other nodes

$$\ell_i = \frac{1}{n} \sum_j d_{ij}$$

Closeness centrality:

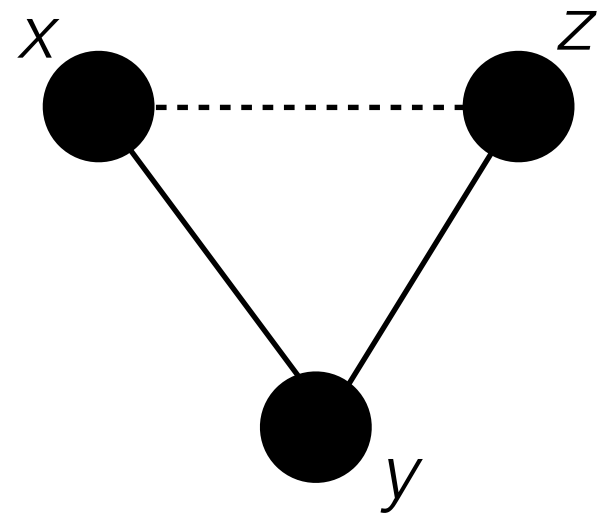
$$C_i = \frac{1}{\ell_i} = \frac{n}{\sum_j d_{ij}}$$

# Interpretation of centrality measures

Centrality measure	Interpretation in social networks
<b>Degree</b>	How many people can this person reach directly?
<b>Eigenvector</b>	How well is this person connected to other well-connected people?
<b>Betweenness</b>	How likely is this person likely to be the most direct route between two people in the network?
<b>Closeness</b>	How fast can this person reach everyone in the network?

# Transitivity and Clustering

- How predictive is the fact that A is friends with B and C of whether B and C are also friends?



The path  $xyz$  is **closed** if the third edge from  $z$  to  $x$  is present.

**Clustering coefficient:**

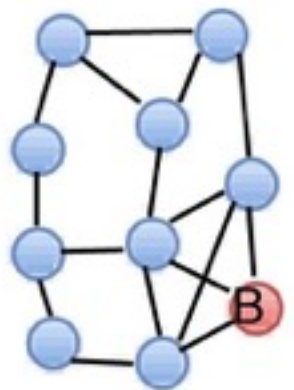
$$C = \frac{(\text{number of triangles}) \times 3}{(\text{number of connected triples})}$$

# Local Clustering

Local clustering for node  $i$ :

$$C_i = \frac{(\text{number of pairs of neighbors of } i \text{ that are connected})}{(\text{number of pairs of neighbors of } i)}$$

- Similar to betweenness centrality
- Can be used to probe for **structural holes**
- Watts-Strogatz “Average clustering” coefficient:



$$C_{WS} = \frac{1}{n} \sum_{i=1}^n C_i$$

# Community Detection

- Separating the network into groups of nodes that are highly connected within groups and sparsely connected between groups.
- Several algorithms exist, each with their own pros and cons.





# Interaction Models on Networks

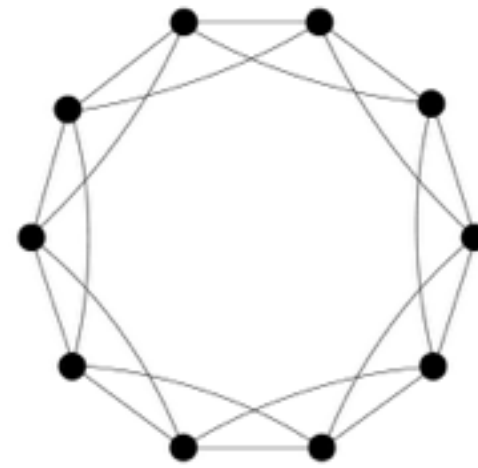
- Epidemics
- Diffusion of innovations or information
- Evolutionary games
- Economic transactions
- Food webs

# Models of Network Architectures

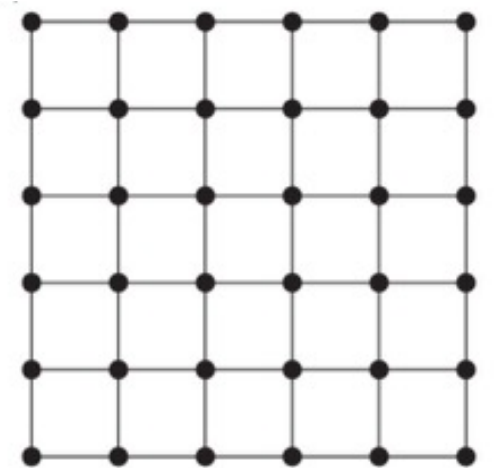
- Regular lattices
- Random networks
- Small-world networks
- Scale-free networks

# Lattices

- Characterized by regular structure
- Easy to model computationally
- Sometimes possible to solve analytically
- Questionable realism



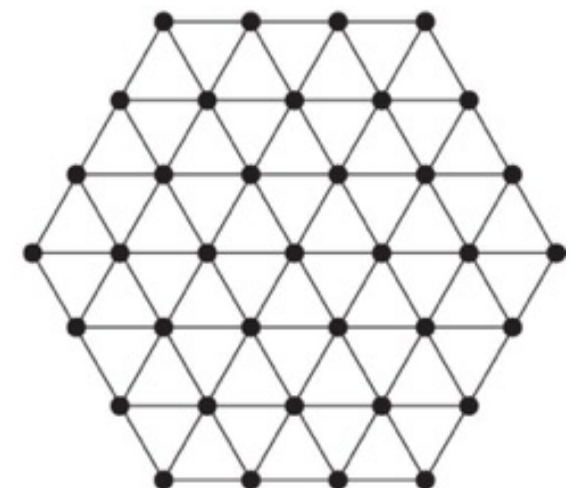
Ring lattice



Square lattice



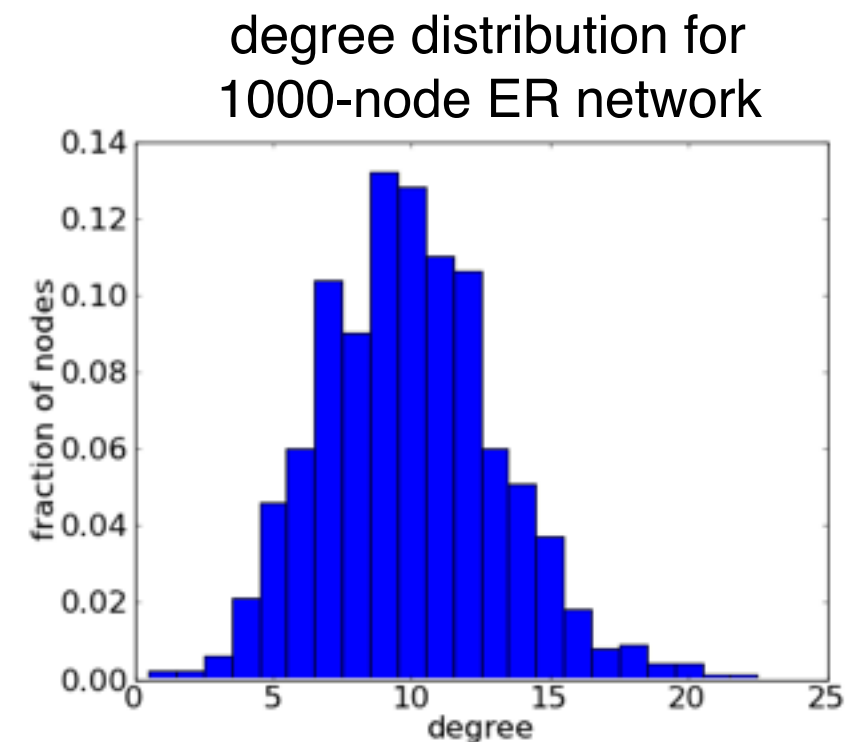
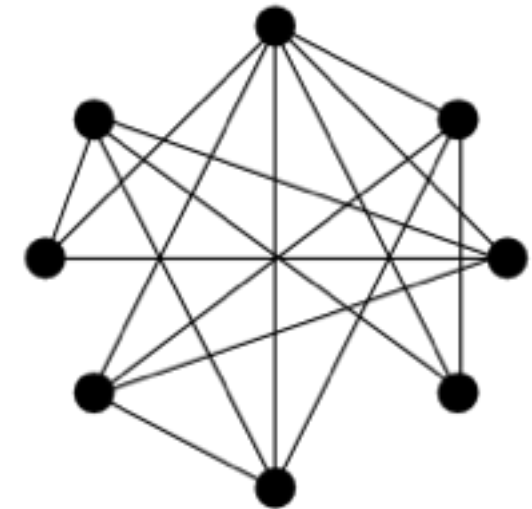
Triangular lattice



Hexagonal lattice

# Random Networks

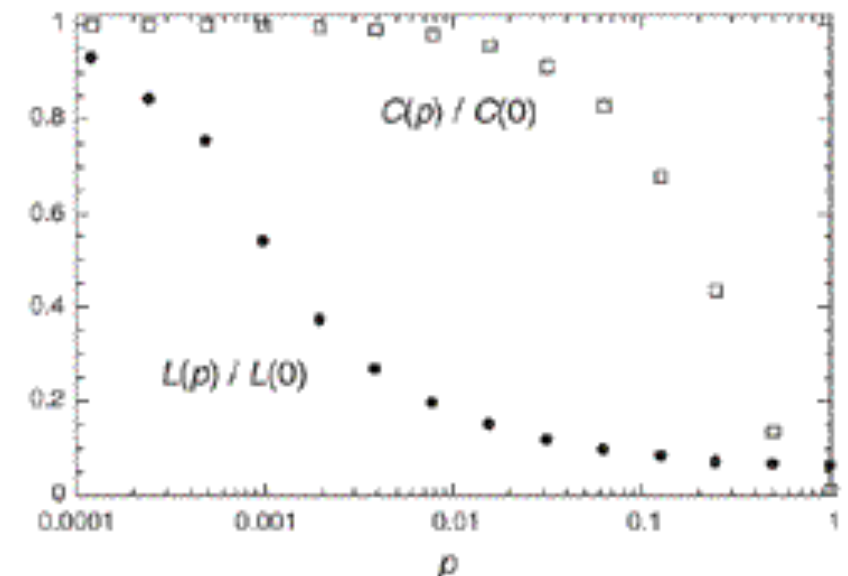
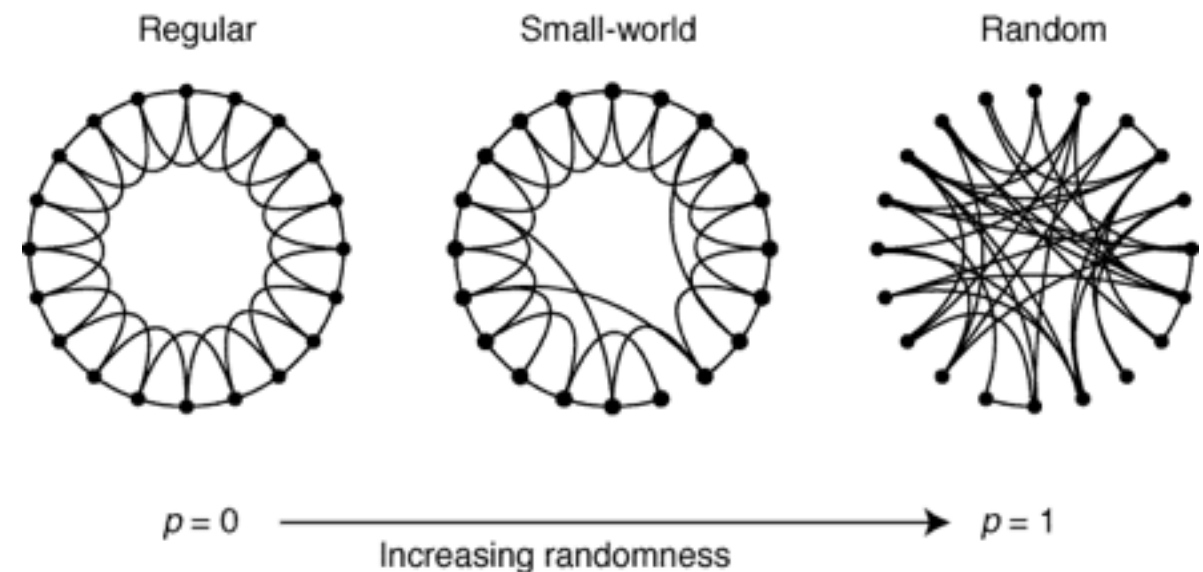
- Introduced by Erdős & Renyi (1959)
- Minimal assumption for a connected population
- Multiple network formation algorithms exist. Example:  $N$  nodes are specified, and each possible edge is added with a fixed probability
- Average degree is predictable, but degree varies between nodes
- Probably not realistic for many systems



# Small-world networks

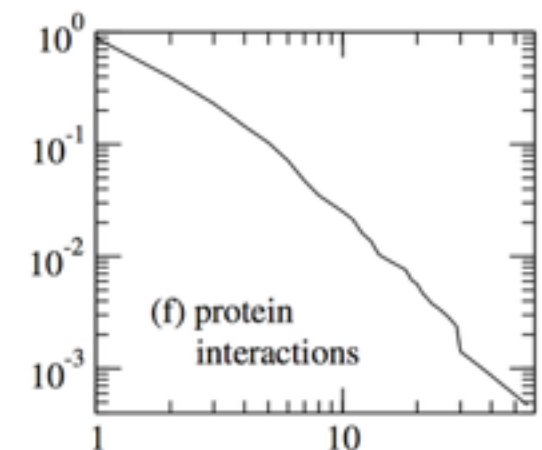
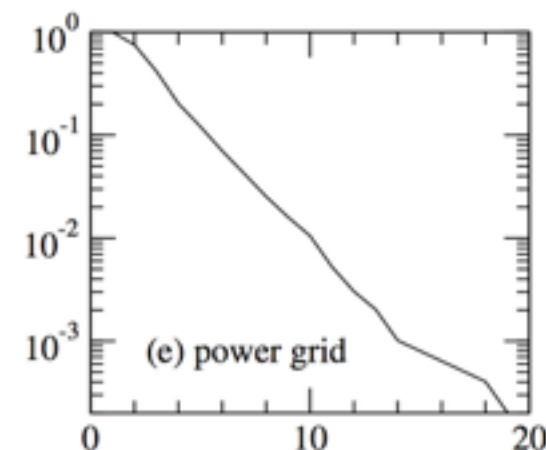
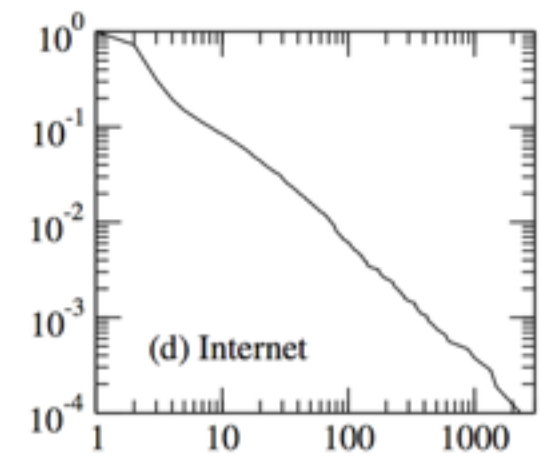
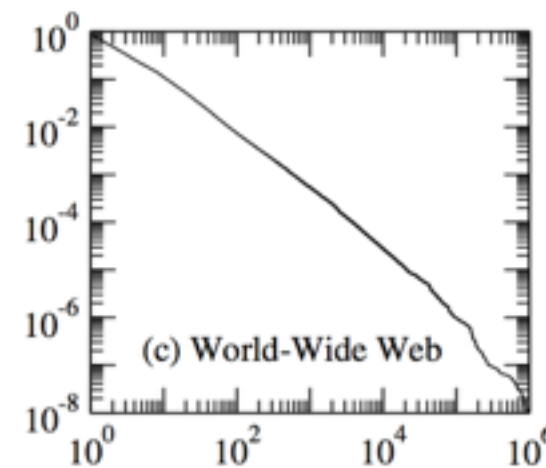
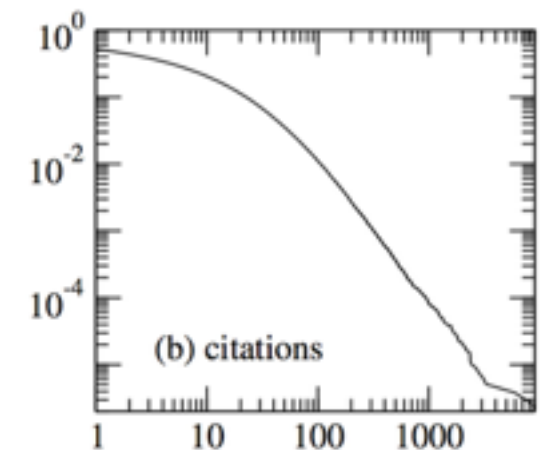
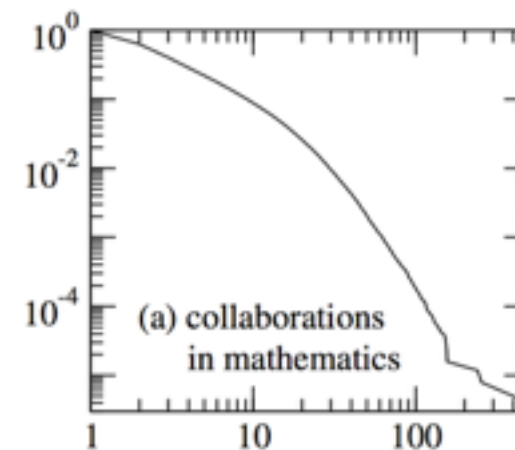
- Introduced by Watts and Strogatz (1998)
- Characterized by high clustering (like lattices) and short path lengths (like random networks)
- Many real world networks share this property:
  - Film actors (IMDB)
  - Power grid nodes and high-voltage transmission lines in Western US
  - Neural network of *C. elegans*
- Fat-tailed degree distribution: overabundance of hubs

$p$  = probability of rewiring edge



# Scale-free networks

- Scale-free: Parts of the network exhibit similar features as the whole network
- Many real-world networks exhibit **power-law degree distributions**
- Few high-degree nodes (hubs), many low-degree nodes



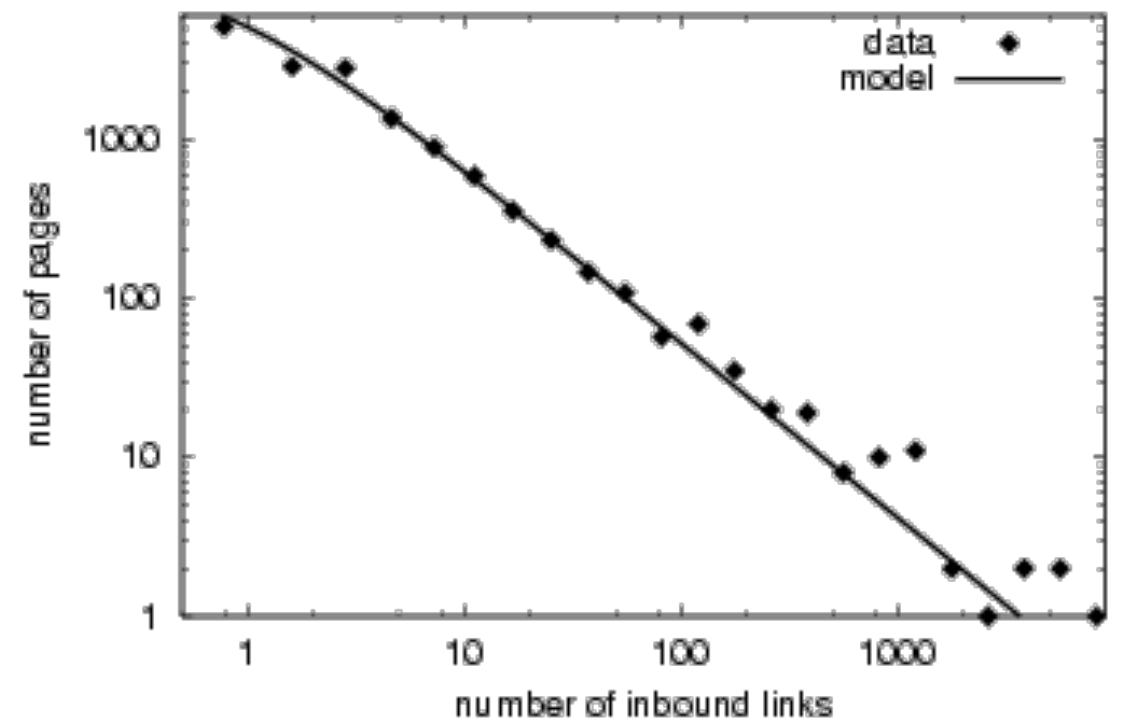
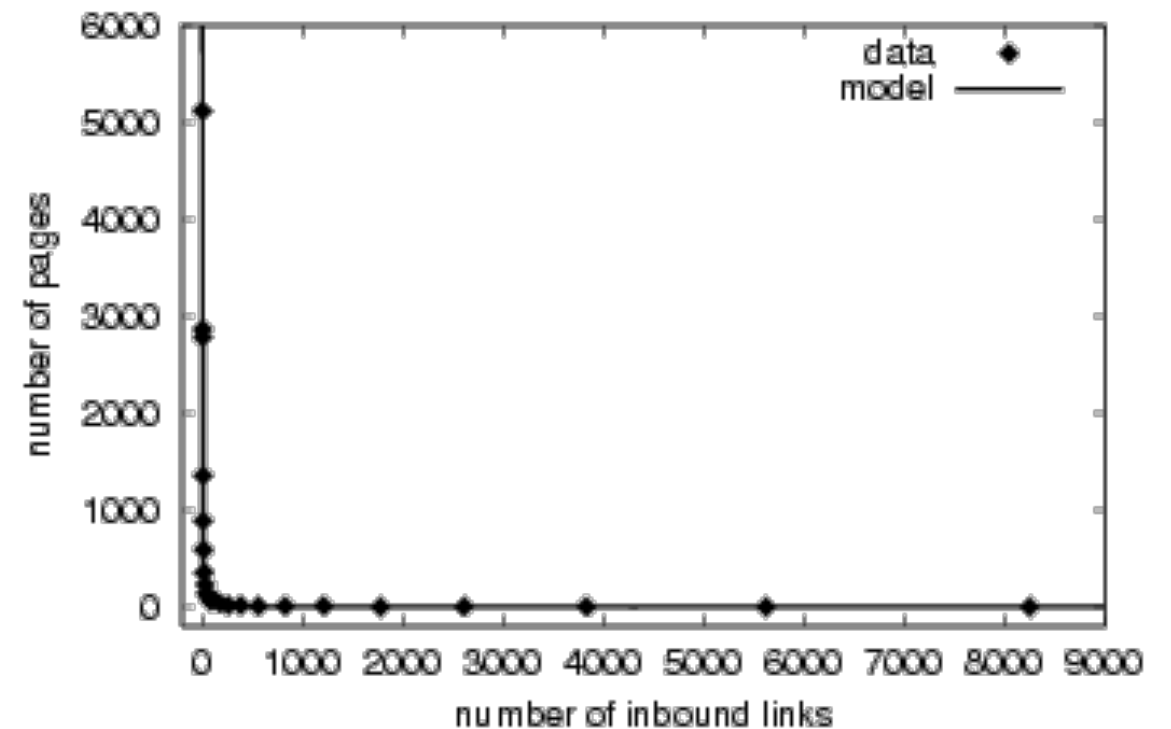
Why do power laws exhibit as straight lines on log-log plots?

$$y = ax^{-k}$$

$$\log y = \log(ax^{-k})$$

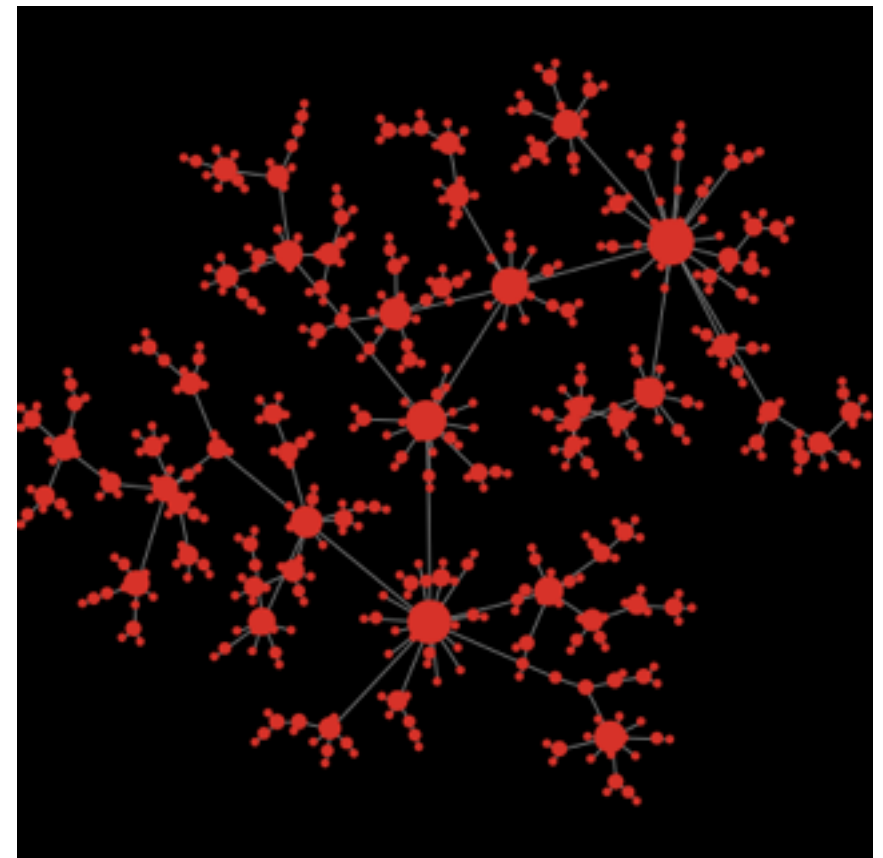
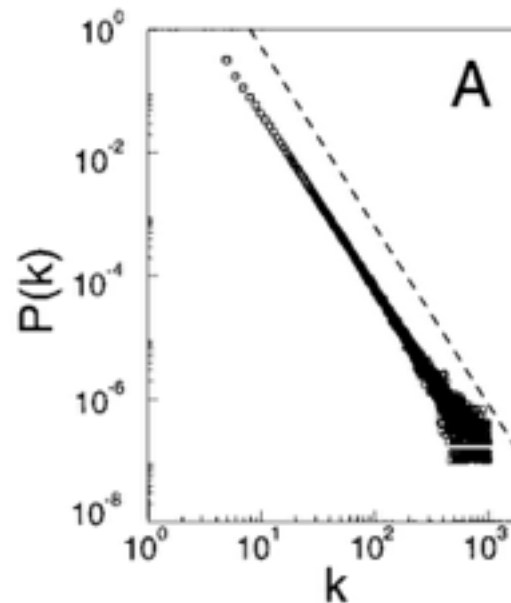
$$= \log a + \log(x^{-k})$$

$$= \log a - k \log x$$



# Preferential Attachment Algorithm

- Barabási & Albert (1999)
- Nodes are added sequentially
- Connectivity is not uniformly random, but preferential
- **Model**
  - Start with  $m_0$  nodes
  - Each time step, add a new node with  $m$  edges, that link to  $m$  existing nodes with a probability proportionate to the current degree of those nodes (relative to all other nodes)
  - “The rich get richer”



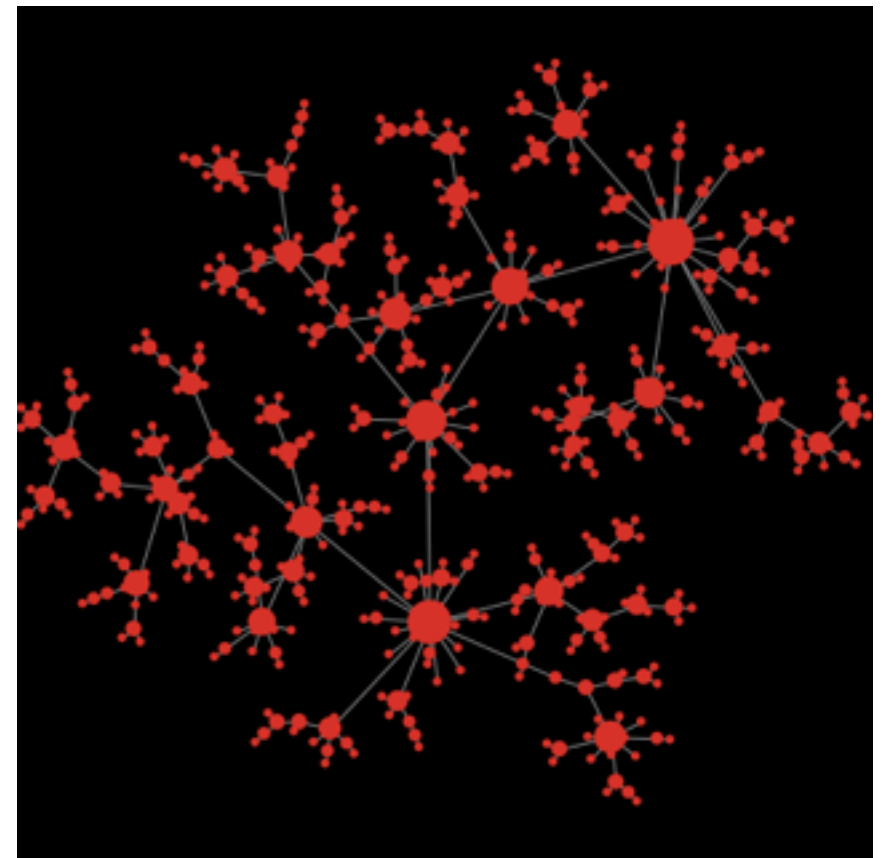
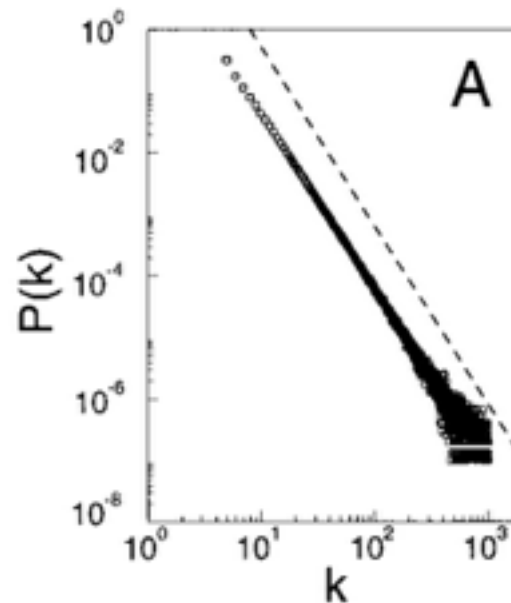






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```

to setup
  clear-all
  set-default-shape turtles "circle"
  ;; make the initial network of two turtles and an edge
  make-node nobody      ;; first node, unattached
  make-node turtle 0    ;; second node, attached to first node
  reset-ticks
end

;;;;;;;;;;;;;
;;; Main Procedures ;;;
;;;;;;;;;;;;;

to go
  ;; new edge is green, old edges are gray
  ask links [ set color gray ]
  make-node find-partner      ;; find partner & use it as attachment
                              ;; point for new node

  tick
  if layout? [ layout ]
end

;; used for creating a new node
to make-node [old-node]
  create-turtles 1
  [
    set color red
    if old-node != nobody
      [ create-link-with old-node [ set color green ]
        ;; position the new node near its partner
        move-to old-node
        fd 8
      ]
  ]
end

;; This code is the heart of the "preferential attachment" mechanism, and acts like
to-report find-partner
  report [one-of both-ends] of one-of links
end

```







Why model?

- Models formalize and scaffold theory development
- Good theory structures the interpretation of data
- Good theory leads to better hypothesis formation

## Surrogates for Theories

**Gerd Gigerenzer**

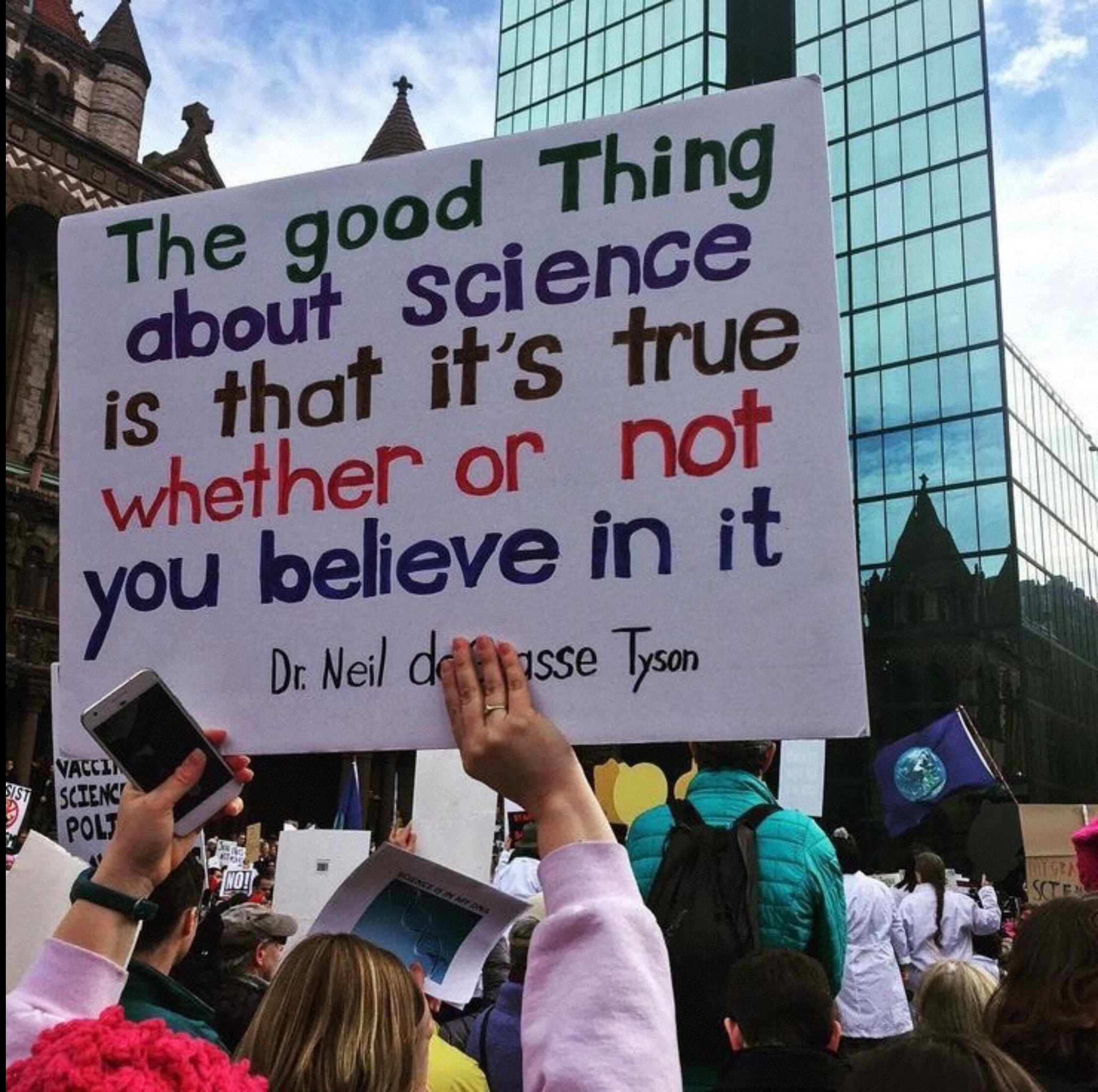
Max Planck Institute for Human Development

Several years ago, I spent a day and a night in a library reading through issues of the *Journal of Experimental Psychology* from the 1920s and 1930s. This was professionally a most depressing experience. Not because these articles were methodologically mediocre. On the contrary, many of them make today's research pale in comparison to their diversity of methods and statistics, their detailed reporting of single-case data rather than mere averages, and their careful selection of trained subjects. And many topics—such as the influence of the gender of the experimenter on the performance of the participants—were of interest then as now. What depressed me was that almost all of this work is forgotten; it does not seem to have left a trace in the collective memory of our profession. It struck me that most of it involved collecting data without substantive theory. Data without theory are like a baby without a parent: their life expectancy is low.



The good Thing  
about science  
is that it's true  
whether or not  
you believe in it

Dr. Neil deGrasse Tyson



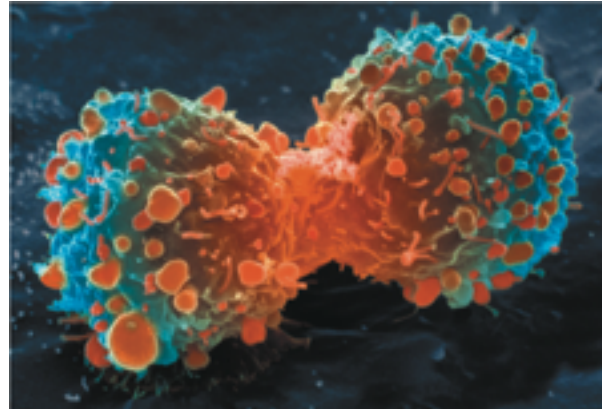


# Counterpoint:

## Oncology

47/53 'landmark' studies did not replicate

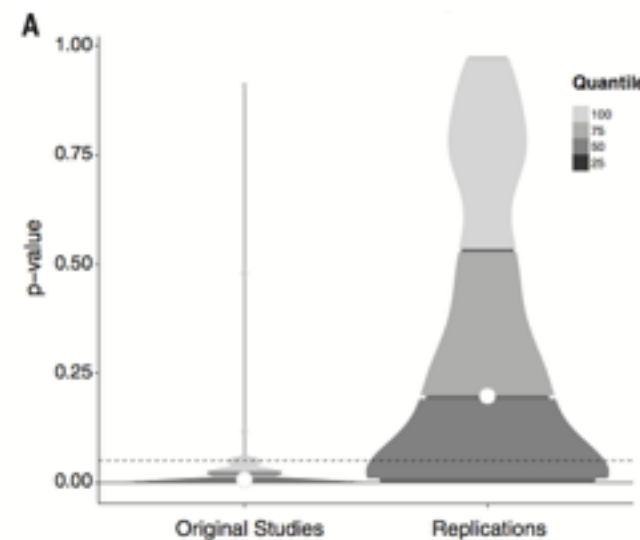
(Begley & Ellis 2012, *Nature*)



## Psychology

61/100 studies in top journals failed to replicate ( $p < .05$ )

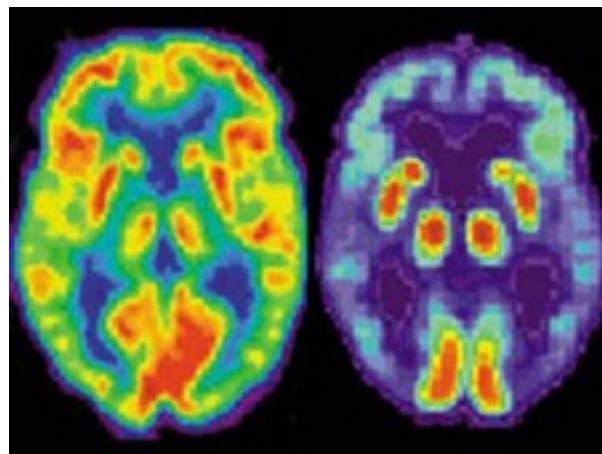
(Open Science Collaboration 2015, *Science*)



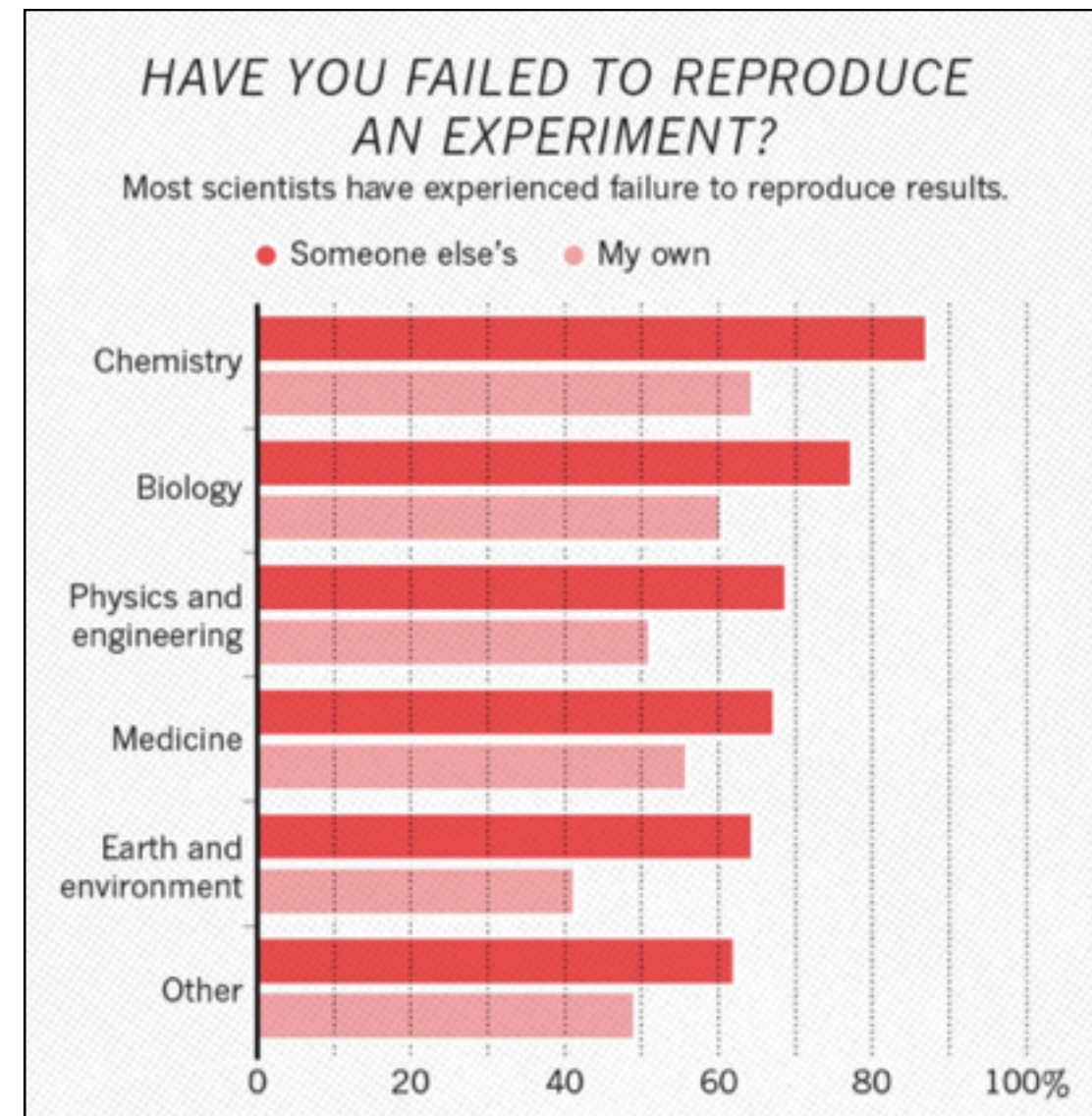
## Neuroscience

Errors in popular statistical methods imply false positive rate of up to 70%

(Eklund et al. 2016, *PNAS*)



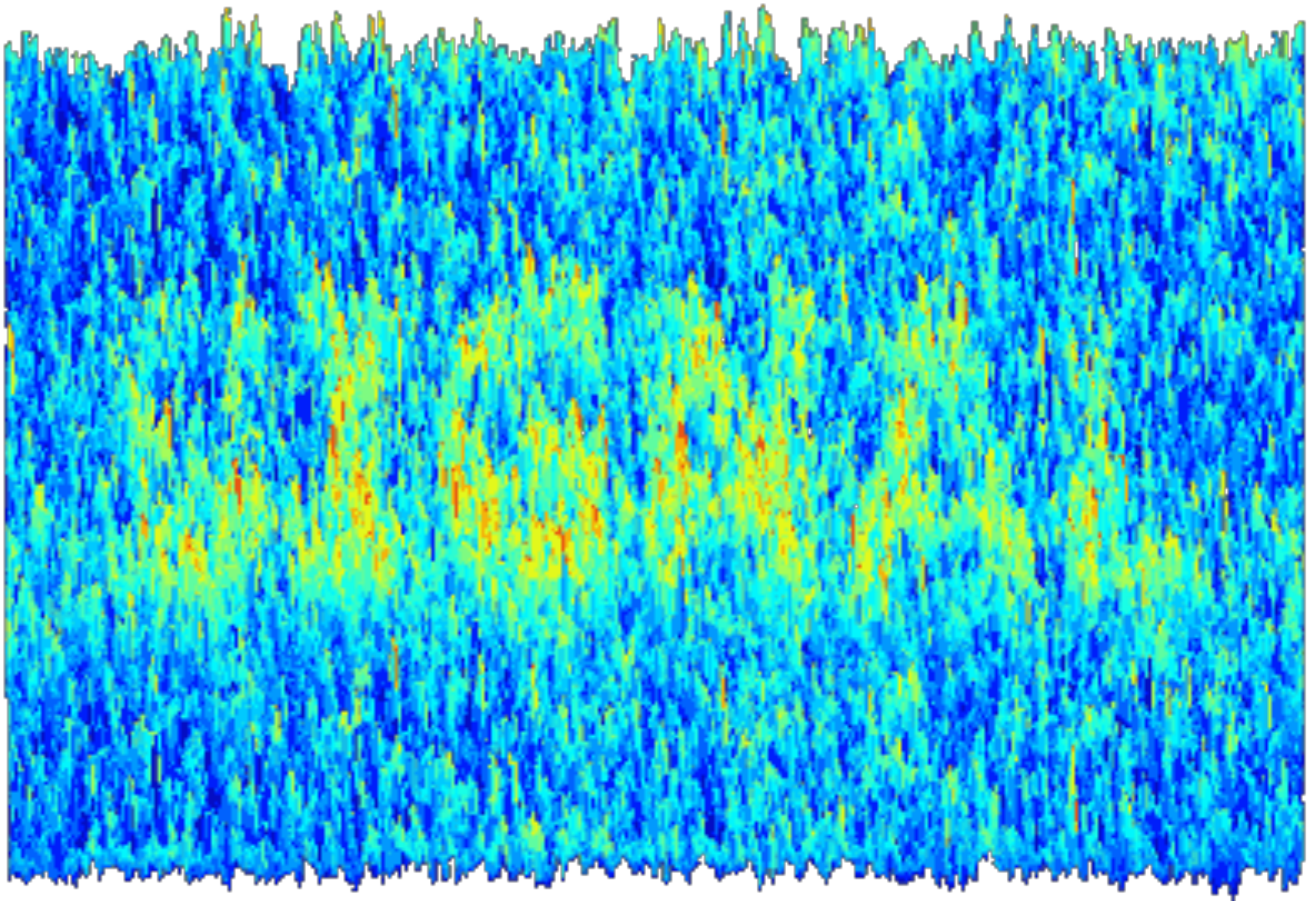
## Most fields?



(Baker 2016, *Nature*)



# Science as Signal Detection for Facts

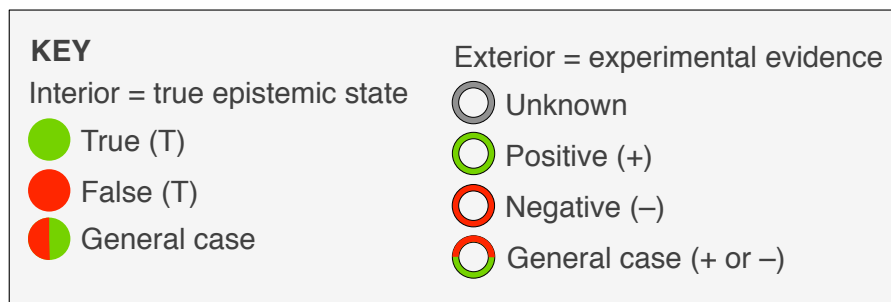
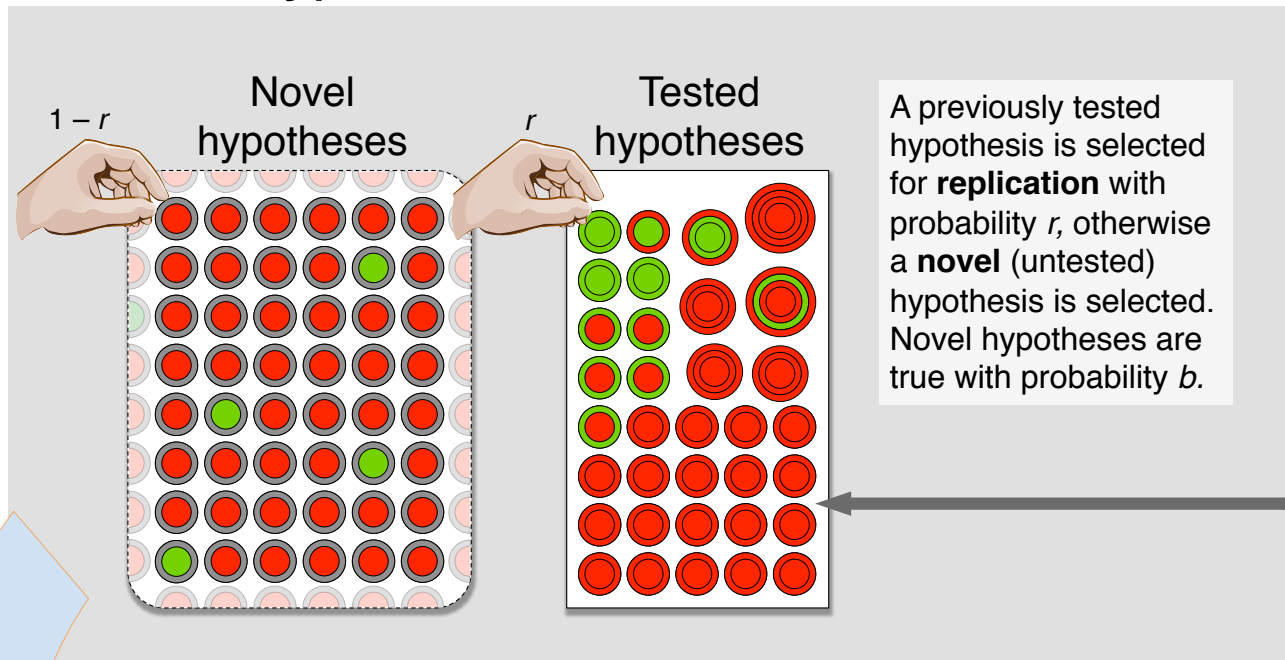


# How do we find facts?

		Real truth of hypothesis		
		T	F	
Probability of result	+	$1 - \beta$	$\alpha$	positive results
	-	$\beta$	$1 - \alpha$	negative results



# 1. Hypothesis Selection



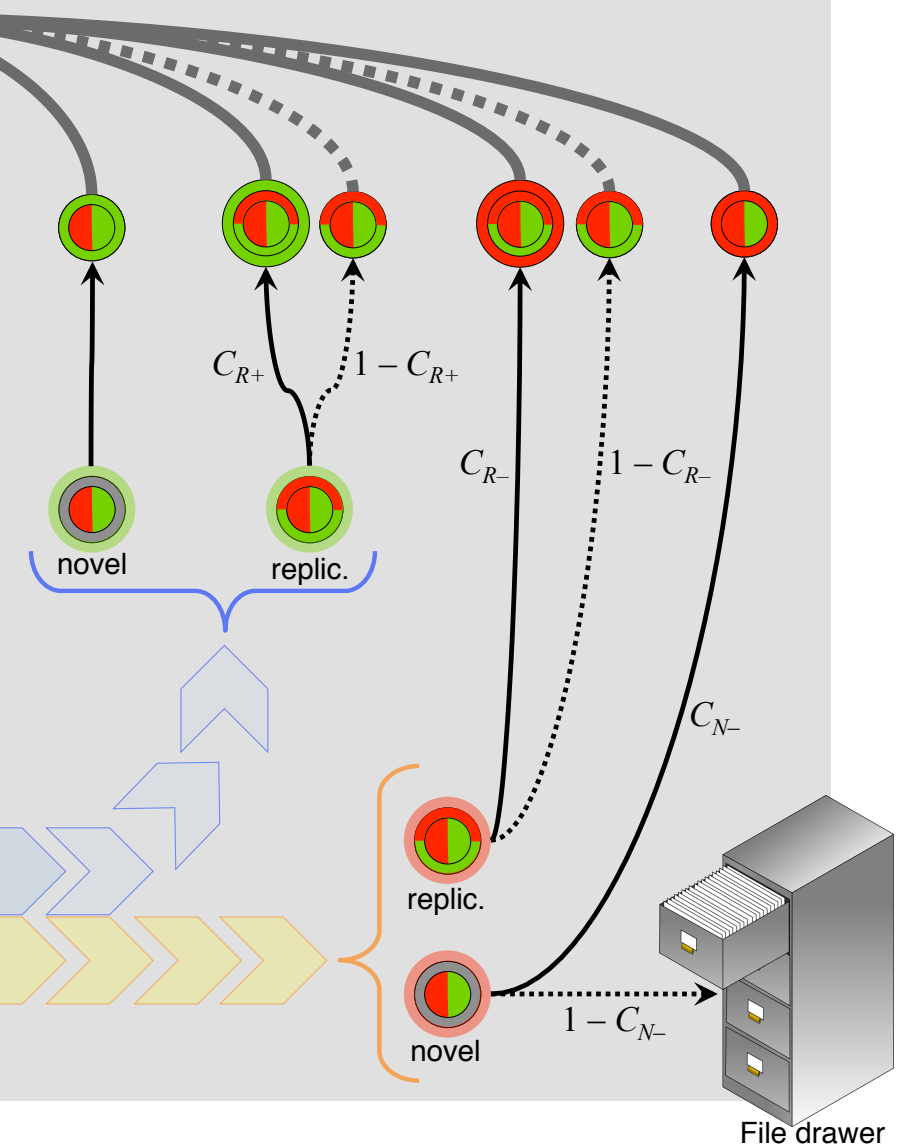
# 2. Investigation

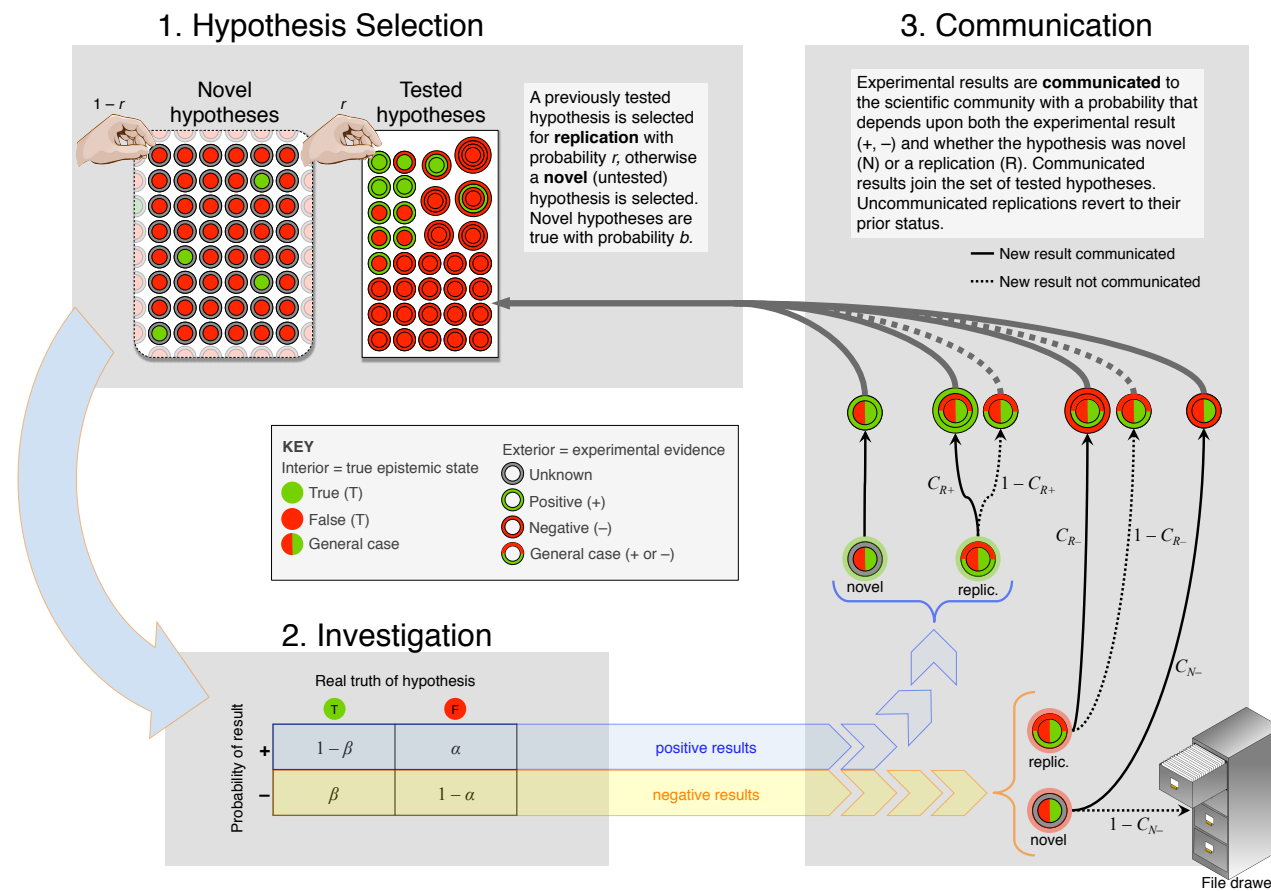
		Real truth of hypothesis		
		T	F	
Probability of result	+	$1 - \beta$	$\alpha$	positive results
	-	$\beta$	$1 - \alpha$	negative results

# 3. Communication

Experimental results are **communicated** to the scientific community with a probability that depends upon both the experimental result (+, -) and whether the hypothesis was novel (N) or a replication (R). Communicated results join the set of tested hypotheses. Uncommunicated replications revert to their prior status.

— New result communicated  
 ..... New result not communicated





*Recursions:*

$$n'_{T,s} = n_{T,s} + anr(-f_{T,s}(c_{R+}(1 - \beta) + c_{R-}\beta) + f_{T,s-1}(1 - \beta)c_{R+} + f_{T,s+1}\beta c_{R-})$$

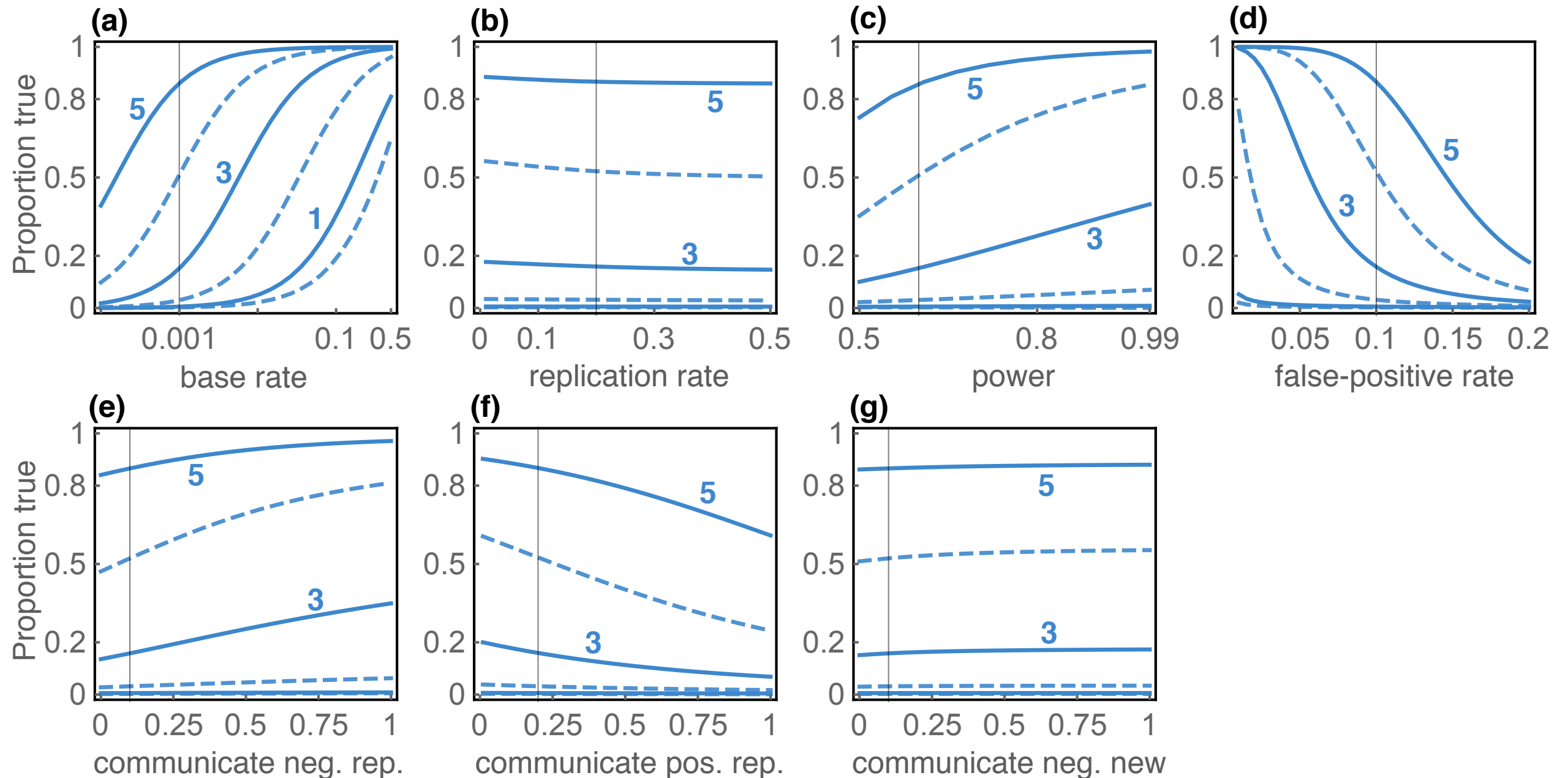
*Solutions:*

$$\hat{p}_{T,s} = b(1 - r) \sum_{m=1}^{\infty} r^{m-1} K(m, (m + s)/2) (1 - \beta)^{\frac{1}{2}(m+s)} \beta^{\frac{1}{2}(m-s)}$$

McElreath R & Smaldino PE (2015) Replication, communication, and the population dynamics of scientific discovery. *PLOS ONE* 10(8):e0136088.

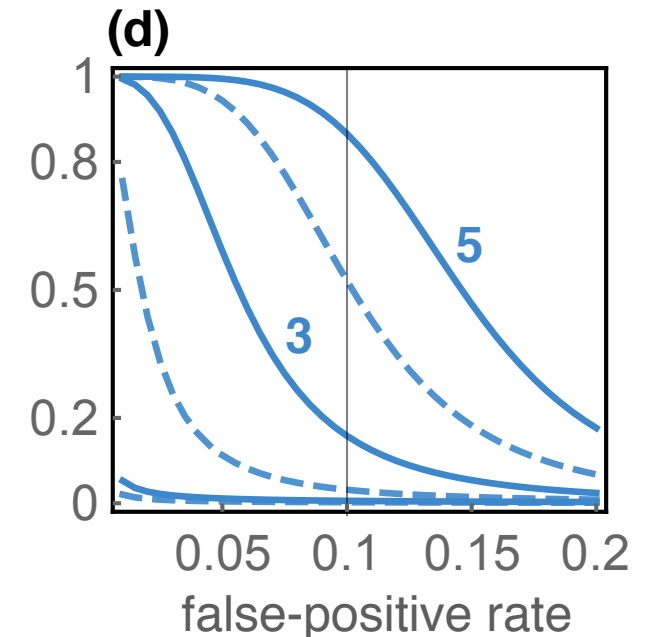
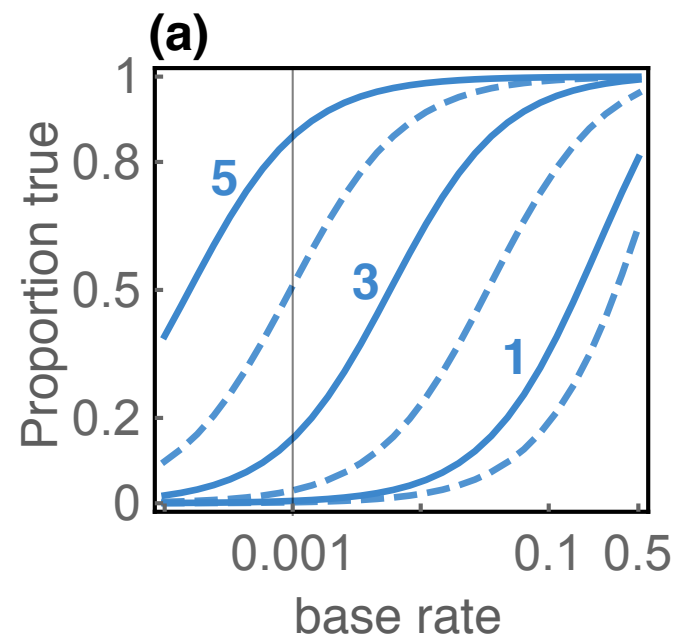


# Proportion true hypotheses at different *numbers of net positive findings*



McElreath R & Smaldino PE (2015) Replication, communication, and the population dynamics of scientific discovery. *PLOS ONE* 10(8):e0136088.

## Proportion true hypotheses at different *numbers of net positive findings*



Base rate and false-positive rate most important factors

“Nothing in biology makes sense  
except in light of evolution”

–Theodosius Dobzhansky (1973)



“All social science research must do some violence to reality in order to reveal simple truths.”

–Lazer & Friedman (2007)

# Turning your idea into a model

- Not a trivial problem
- Look for existing solutions
- Get creative
- Keep it simple (KISS)
- Solicit feedback
- Remember Hofstadter's Law

